## Oracle ${ }^{\circledR}$ Developer Studio 12.6:

 Performance Library User's Guide
## Part No: E77802

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## Using This Documentation

- Overview - Describes how to use the unique extensions and features included with the Oracle Developer Studio Performance Library subroutines that are supported by the Oracle Developer Studio Fortran 95, C++, and C compilers.
- Audience - Application developers, system developers, architects, support engineers.
- Required knowledge - You should have a working knowledge of the Fortran or C language, basic knowledge of numerical analysis, and some understanding of the base LAPACK and BLAS libraries available from Netlib (http://www. netlib.org).


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## Introduction to Oracle Developer Studio Performance Library


#### Abstract

Oracle Developer Studio Performance Library is a set of optimized, high-speed mathematical subroutines for solving linear algebra and other numerically intensive problems. Oracle Developer Studio Performance Library is based on a collection of public domain applications mostly available from Netlib at http://www. netlib.org. These public domain applications have been enhanced and bundled together as the Oracle Developer Studio Performance Library.

This document explains the Oracle-specific enhancements to the base applications available from Netlib. Reference material describing the base routines is available from Netlib and the Society for Industrial and Applied Mathematics (SIAM). Additionally, the Oracle Developer Studio 12.6 Performance Library's public functions and subroutines are documented in detail in Section 3P of the Oracle Developer Studio manual pages. To see information about the 3P man pages, type man $-s 3 p$ intro. For more information about adding the Oracle Developer Studio man page path to your MANPATH variable, see "Setting Up Access to the Developer Tools and Man Pages" in Oracle Developer Studio 12.6: Installation Guide.


## Libraries Included With Oracle Developer Studio Performance Library

The Performance Library contains enhanced versions of the following standard libraries:

- LAPACK version 3.6.1 - For solving linear algebra problems.
- BLAS1 (Basic Linear Algebra Subprograms) - For performing vector-vector operations.
- BLAS2 - For performing matrix-vector operations.
- BLAS3 - For performing matrix-matrix operations.
- Netlib Sparse-BLAS - performing sparse vector operations
- NIST Sparse-BLAS 0.5 - performing fundamental sparse matrix operations
- SuperLU 3.0 - solving sparse linear systems of equations
- Sparse Solver - direct sparse solver routines
- FFTPACK - performing fast Fourier transform
- VFFTPACK - performing vectorized fast Fourier transform
- XBLAS - extra precise basic linear algebra subroutines
- Other Routines - transpose, Convolution, correlation and sort

Note - LINPACK has been removed from Oracle Developer Studio Performance Library. LAPACK version 3.6.1 supersedes LINPACK and all previous versions of LAPACK. If the LINPACK routines are still needed, you can obtain the LINPACK library and documentation from http://www.netlib.org.

Oracle Developer Studio Performance Library is available in both static and dynamic library forms. There are optimized SPARC versions for sparcvis, sparcvis2, and sparcfmaf and advanced architectures on the Oracle Solaris 11 and Oracle Linux operating systems. There are also optimized versions for x86/x64 architectures on Oracle Solaris 11 systems, along with Oracle Linux systems. All versions have support for parallel programming on multiprocessor platforms. See the Oracle Developer Studio 12.6: Release Notes for details.

Oracle Developer Studio Performance Library LAPACK routines have been compiled with a Fortran 95 compiler and remain compatible with the Netlib LAPACK version 3.6.1 library. The Performance Library versions of these routines perform the same operations as the Fortran callable routines and have the same interface as the standard Netlib versions.

LAPACK contains driver, computational, and auxiliary routines. The Performance Library does not support the auxiliary routines, because auxiliary routines can be changed or be removed from LAPACK without notice. Because the auxiliary routines are not supported, they are not documented in this user guide or in the 3P section man pages.

Many auxiliary routines contain LA as the second and third characters in the routine name; however, some do not. Appendix B of the LAPACK User's Guide, Third Edition (http://www. netlib.org/lapack/lug/) contains a list of auxiliary routines.

## About Netlib

Netlib is an online repository of mathematical software, papers, and databases maintained by AT\&T Bell Laboratories, the University of Tennessee, Oak Ridge National Laboratory, and professionals from around the world.

Netlib provides many libraries, in addition to the libraries used in Oracle Developer Studio Performance Library. While some of these libraries can appear similar to libraries used with the Performance Library, they can be different from, and incompatible with the Performance Library.

Using routines from other libraries can produce compatibility problems, not only with Oracle Developer Studio Performance Library routines, but also with the base Netlib LAPACK routines. When using routines from other libraries, refer to the documentation provided with those libraries.

For example, Netlib provides a CLAPACK library, but the CLAPACK interfaces differ from the C interfaces included with Oracle Developer Studio Performance Library. A LAPACK 90 library package is also available on Netlib. The LAPACK 90 library contains interfaces that differ from the Oracle Developer Studio Performance Library Fortran 95 interfaces and the Netlib LAPACK version 3.6.1 interfaces. If using LAPACK 90, refer to the documentation provided with that library.

For the base libraries supported by Oracle Developer Studio Performance Library, Netlib provides detailed information that can supplement this user's guide. The LAPACK User's Guide, Third Edition (http://www.netlib.org/lapack/lug/) describes LAPACK algorithms and how to use the routines, but it does not describe the Oracle Developer Studio Performance Library extensions made to the base routines.

## Related Documentation

The LAPACK User's Guide is the official reference for the base LAPACK version 3.6.1 routines. An online version of the LAPACK Users' Guide is available at http://www. netlib. org/lapack/lug/, and the printed version is available from the Society for Industrial and Applied Mathematics (SIAM) http://www. siam.org.

Oracle Developer Studio Performance Library routines contain performance enhancements, extensions, and features not described in the LAPACK Users' Guide. However, because Oracle Developer Studio Performance Library maintains compatibility with the base LAPACK routines, the LAPACK Guide can be used as a reference for the LAPACK routines and the Fortran interfaces.

Note - LINPACK has been removed from the Oracle Developer Studio Performance Library. The LINPACK libraries and documentation are still available from http://www. netlib.org.

See the following locations for information describing the performance library routines that form the basis of the Oracle Developer Studio Performance Library.

```
LAPACK version 3.6.1 http://www.netlib.org/lapack/
BLAS, levels 1 through 3 http://www.netlib.org/blas/
FFTPACK version 4
    http://www.netlib.org/fftpack/
```

VFFTPACK version 2.1

Sparse BLAS

NIST (National Institute of Standards and Technology) Fortran Sparse BLAS

SuperLU version 3.0
http://www.netlib.org/vfftpack/
http://www.netlib.org/sparse-blas/index.html
http://math.nist.gov/spblas/
http://crd.lbl.gov/~xiaoye/SuperLU/

## Oracle Developer Studio Performance Library Features

Oracle Developer Studio Performance Library routines can increase application performance on both serial and multiprocessor (MP) platforms, because the serial speed of many Performance Library routines has been increased, and many routines have been parallelized. Oracle Developer Studio Performance Library routines also have SPARC, AMD, and Intel specific optimizations that are not present in the base Netlib libraries.

Oracle Developer Studio Performance Library provides the following optimizations and extensions to the base Netlib libraries:

- Extensions that support Fortran 95 and C language interfaces
- Fortran 95 language features, including type independence and compile time checking
- Consistent API across the different libraries in the Performance Library
- Compatibility with LAPACK 1.0, 2.0, 3.0, 3.1.1, 3.3.1, 3.4.2 and 3.5.0 libraries
- Increased performance, and in some cases, greater accuracy
- Optimizations for specific SPARC and x86/x64 instruction set architectures
- Support for 64-bit enabled Oracle Solaris and Linux operating environments
- Support for parallel processing compiler options for SPARC and x86/x64 platforms
- Support for multiple processor hardware options


## Mathematical Routines

Oracle Developer Studio Performance Library routines are used to solve the following types of linear algebra and numerical problems:

- Elementary vector and matrix operations - Vector and matrix products; plane rotations; 1, 2-, and infinity-norms; rank-1, 2, k, and 2k updates
- Linear systems - Solve full-rank systems, compute error bounds, solve Sylvester equations, refine a computed solution, equilibrate a coefficient matrix
- Least squares - Full-rank, generalized linear regression, rank-deficient, linear equality constrained
- Eigenproblems - Eigenvalues, generalized eigenvalues, eigenvectors, generalized eigenvectors, Schur vectors, generalized Schur vectors
- Matrix factorizations or decompositions - SVD, generalized SVD, QL and LQ, QR and RQ, Cholesky, LU, Schur, LDL $^{\mathrm{T}}$, UDU ${ }^{\mathrm{T}}$, and CS Decomposition
- Support operations - Condition number, in-place or out-of-place transpose, inverse, determinant, inertia, extra-precise iterative refinement
- Sparse matrices - Solve symmetric, structurally symmetric, and unsymmetric coefficient matrices using direct methods and a choice of fill-reducing ordering algorithms, and userspecified orderings
- Convolution and correlation in one and two dimensions
- Fast Fourier transforms, Fourier analysis and Fourier synthesis, cosine and quarter-wave cosine transforms, cosine and quarter-wave sine transforms
- Complex vector FFTs and FFTs in two and three dimensions
- Sorting operations
- CBLAS Interface


## Compatibility With Previous LAPACK Versions

The Oracle Developer Studio Performance Library routines that are based on LAPACK support the expanded capabilities and improved algorithms in LAPACK 3.6.1, but are completely compatible with LAPACK 1.0, LAPACK 2.0, LAPACK 3.0, LAPACK 3.1.1, LAPACK 3.3.1, LAPACK 3.4.2 and LAPACK 3.5.0 libraries. Maintaining compatibility with previous LAPACK versions:

- Reduces linking errors due to changes in subroutine names or argument lists.
- Ensures results are consistent with results generated with previous LAPACK versions.
- Minimizes programs terminating due to differences between argument lists.


## Getting Started With Oracle Developer Studio Performance Library

This section shows the most basic compiler options used to compile an application that uses the Oracle Developer Studio Performance Library routines.

To use the Oracle Developer Studio Performance Library, type one of the following commands.
On x86/x64 and SPARC platforms:
my_system\% f95 -dalign my_file.f -library=sunperf
On SPARC platforms:
my_system\% cc -xmemalign=8s my_file.c -library=sunperf
my_system\% CC -xmemalign=8s my_file.cpp -library=sunperf

On x86/64 platforms, -xmemalign=8s is ignored and therefore can be omitted:
my_system\% cc my_file.c -library=sunperf
my_system\% CC my_file.cpp -library=sunperf
To link with the Oracle Developer Studio Performance Library statically, add -staticlib=sunperf to the command line.

Because Oracle Developer Studio Performance Library routines are compiled with -dalign, this option should be used for compilation of all Fortran files if any routine in the program makes an Oracle Developer Studio Performance Library call. On SPARC platforms, C and C++ user code that calls Oracle Developer Studio Performance Library routines should be compiled with option -xmemalign=8s. If -xmemalign=8s cannot be used, enabling trap 6 is a low performance workaround that allows misaligned data. See "Enabling Trap 6 on SPARC Platforms" on page 17 for more details.

While there are no data alignment restrictions on x86/x64 platforms, misaligned data might require extra instructions to properly handle memory transfers, which in turn can cause poor performance.

The -library=sunperf option includes additional compiler and system libraries such as the Fortran run-time and micro-tasking library and sets run-time search paths for the resulting executable or shared library.

To summarize, use the following options:

- -dalign on all Fortran files at compile time.
-xmemalign=8s on SPARC platforms, or enable trap 6
- The same command line options for compiling and linking
- -library=sunperf or -library=sunperf -staticlib=sunperf

See "About Compiling" on page 30 and Chapter 4, "Parallel Processing" for additional options that optimize application performance.

## V Enabling Trap 6 on SPARC Platforms

On SPARC platforms where data misalignment can cause failure, if an application cannot be compiled using -dalign or -xmemalign=8s, enable trap 6 to provide a handler for misaligned data. To enable trap 6 on SPARC platforms, do the following:

1. Place this assembly code in a file called trap6_handler.s.
```
.global trap6_handler_
.text
.align 4
trap6_handler_:
    retl
ta 6
```


## 2. Assemble trap6_handler.s.

my_system\% fbe trap6_handler.s
fbe is the command that will create object files from assembly language source files.
The first parallelizable subroutine invoked from Oracle Developer Studio Performance Library will call a routine named trap6_handler_. If a trap6_handler_ is not specified, Oracle Developer Studio Performance Library will call a default handler that does nothing. Not supplying a handler for any misaligned data will cause a trap that will be fatal.
3. Include trap6_handler.o on the command line.
my_system\% f95 any.f trap6_handler.o -library=sunperf

## Using Oracle Developer Studio Performance Library

This chapter describes using the Oracle Developer Studio Performance Library to improve the execution speed of applications written in Fortran 95 or C. The performance of many applications can be increased by using Oracle Developer Studio Performance Library without making source code changes or recompiling. However, some modifications to applications might be required to gain peak performance with Oracle Developer Studio Performance Library.

## Improving Application Performance

The following sections describe ways of using Oracle Developer Studio Performance Library routines without making source code changes or recompiling.

## Replacing Routines With Oracle Developer Studio Performance Library Routines

Many applications use one or more of the base Netlib libraries, such as LAPACK or BLAS. Because Oracle Developer Studio Performance Library maintains the same interfaces and functionality of these libraries, base Netlib routines can be replaced with Oracle Developer Studio Performance Library routines. Application performance is increased, because Oracle Developer Studio Performance Library routines can be faster than the corresponding Netlib routines or similar routines provided by other vendors.

## Improving Performance of Other Libraries

Many commercial math libraries are built around a core of generic BLAS and LAPACK routines. When an application has a dependency on proprietary interfaces in another library that
prevents the library from being completely replaced, the BLAS and LAPACK routines used in that library can be replaced with the Oracle Developer Studio Performance Library BLAS and LAPACK routines. Because replacing the core routines does not require any code changes, the proprietary library features can still be used, and the other routines in the library can remain unchanged.

## Using Tools to Restructure Code

Some libraries that do not directly use Oracle Developer Studio Performance Library routines can be modified by using automatic code restructuring tools that replace existing code with Oracle Developer Studio Performance Library code. For example, a source- to- source conversion tool can replace existing BLAS code structures with calls to the Oracle Developer Studio Performance Library BLAS routines. These conversion tools can also recognize many user written matrix multiplications and replace them with calls to the matrix multiplication subroutine in Oracle Developer Studio Performance Library.

## Fortran Interfaces

Oracle Developer Studio Performance Library contains f95 interfaces and legacy f77 interfaces for maintaining compatibility with the standard LAPACK and BLAS libraries and existing codes. Oracle Developer Studio Performance Library f95 and legacy f77 interfaces use the following conventions:

- All arguments are passed by reference.
- Types of arguments must be consistent within a call. For example, do not mix REAL*8 and REAL*4 parameters in the same call.
- Arrays are stored columnwise.
- Indices are based at one, in keeping with standard Fortran practice.

Keep in mind the following information when calling Oracle Developer Studio Performance Library routines:

- Do not prototype the subroutines with the Fortran 95 INTERFACE statement. Use the USE SUNPERF statement instead.
- Do not use -ext_names=plain to compile routines that call routines from Oracle Developer Studio Performance Library.


## Fortran sunPerf Module for Use With Fortran 95

Oracle Developer Studio Performance Library provides a Fortran module for additional ease-ofuse features with Fortran 95 programs. To use this module, include the following line in Fortran 95 codes.

USE SUNPERF

USE statements must precede all other statements in the code, except for the PROGRAM or SUBROUTINE statement.

The SUNPERF module contains interfaces that simplify the calling sequences and provides the following features:

- Type Independence - Oracle Developer Studio Performance Library supports interfaces where the type of the data arguments will automatically be recognized, eliminating the need for type-dependent prefixes (S, D, C, or Z). In the FORTRAN 77 routines, the type must be specified as part of the routine name. For example, DGEMM is a double precision matrix multiply and SGEMM is a single precision matrix multiply. When calling GEMM with the Fortran 95 interfaces, Fortran will infer the type from the arguments that are passed. Passing single-precision arguments to GEMM gets results that are equivalent to specifying SGEMM, and passing double-precision arguments gets results that are equivalent to DGEMM. For example, $\operatorname{CALL} \operatorname{DSCAL}(20,5.26 \mathrm{D} 0, \mathrm{X}, 1)$ could be changed to $\operatorname{CALL} \operatorname{SCAL}(20,5.26 \mathrm{D} 0, \mathrm{X}, 1)$.
- Compile-Time Checking - In FORTRAN 77, it is generally impossible for the compiler to determine what arguments should be passed to a particular routine. In Fortran 95, the USE SUNPERF statement allows the compiler to determine the number, type, size, and shape of each argument to each Oracle Developer Studio Performance Library routine. It can check the calls against the expected value and display errors during compilation.
- 64-bit Integer Support - When using the 64-bit interfaces provided with Oracle Developer Studio Performance Library, integer arguments must be promoted to 64-bits, and the routine name must be modified by appending _64 to the routine name. With the SUNPERF module, 64-bit integers will automatically be recognized, which eliminates the need for appending _64 to the routine name, as shown in the following code example:

```
SUBROUTINE SUB(N,ALPHA,X,Y)
USE SUNPERF
INTEGER(8) N
REAL(8) ALPHA, X(N), Y(N)
! EQUIVALENT TO DAXPY_64(N,ALPHA,X,1_8,Y,1_8)
CALL DAXPY(N,ALPHA,X,1_8,Y,1_8)
```

For a detailed description of using the Oracle Developer Studio Performance Library 64-bit interfaces, see "Compiling Code for a 64-Bit Enabled Operating Environments" on page 31.

Because the sunperf.mod file is compiled with - dalign, any code that contains the USE SUNPERF statement must be compiled with -dalign. The following error occurs if the code is not compiled with -dalign.

```
use sunperf
    "test_code.f", Line = 2, Column = 11: ERROR: Procedure "SUNPERF" and this
compilation must both be compiled with -dalign, or without -dalign.
```


## Fortran Examples

To increase the performance of single processor applications, identify code constructs in an application that can be replaced by calls to Oracle Developer Studio Performance Library routines. Performance of multiprocessor applications can be increased by identifying opportunities for parallelization.

To increase application performance by modifying code to use Oracle Developer Studio Performance Library routines, identify blocks of code that exactly duplicate the capability of a Oracle Developer Studio Performance Library routine. The following code example is the matrix-vector product $y \leftarrow A x+y$, which can be replaced with the DGEMV subroutine.

```
DO I = 1, N
    DO J = 1, N
        Y(I) = Y(I) + A(I,J)* X(J)
    END DO
END DO
```

In other cases, a block of code can be equivalent to several Oracle Developer Studio Performance Library calls or contain portions of code that can be replaced with calls to Oracle Developer Studio Performance Library routines. Consider the following code example.

```
DO I = 1, N
    IF (V2(I,K) .LT. 0.0) THEN
        V2(I,K) = 0.0
    ELSE
        DO J = 1, M
            X(J,I) = X(J,I) + Vl(J,K)* V2(I,K)
        END DO
    END IF
```

END DO
The code example can be rewritten to use the Oracle Developer Studio Performance Library routine DGER, as shown here.

```
DO I = 1, N
    IF (V2(I,K) .LT. 0.0) THEN
        V2(I,K) = 0.0
    END IF
END DO
CALL DGER (M, N, 1.0D0, X, LDX, Vl(l,K), 1, V2(1,K), 1)
```

The same code example can also be rewritten using Fortran 95 specific statements, as shown here.

```
WHERE (V(1:N,K) .LT. 0.0) THEN
    V(1:N,K) = 0.0
END WHERE
CALL DGER (M, N, 1.0D0, X, LDX, Vl(l,K), 1, V2(1,K), 1)
```

Because the code to replace negative numbers with zero in V2 has no natural analog in Oracle Developer Studio Performance Library, that code is pulled out of the outer loop. With that code removed to its own loop, the rest of the loop is a rank-1 update of the general matrix $x$ that can be replaced with the DGER routine from BLAS.

The amount of performance increase can also depend on the data the Oracle Developer Studio Performance Library routine uses. For example, if V2 contains many negative or zero values, the majority of the time might not be spent in the rank-1 update. In this case, replacing the code with a call to DGER might not increase performance.

Evaluating other loop indexes can affect the Oracle Developer Studio Performance Library routine used. For example, if the reference to K is a loop index, the loops in the code sample shown above might be part of a larger code structure, where the loops over DGEMV or DGER could be converted to some form of matrix multiplication. If so, a single call to a matrix multiplication routine can increase performance more than using a loop with calls to DGER.

Because all Oracle Developer Studio Performance Library routines are MT-safe (multithread safe), using the auto-parallelizing compiler to parallelize loops that contain calls to Oracle Developer Studio Performance Library routines can increase performance on multiprocessor platforms.

An example of combining a Oracle Developer Studio Performance Library routine with an auto-parallelizing compiler parallelization directive is shown in the following code example.

C\$PAR DOALL
DO I = 1, $N$

```
            CALL DGBMV ('No transpose', N, N, ALPHA, A, LDA,
\$ B(l,I), 1, BETA, C(l,I), 1)
END DO
```

Oracle Developer Studio Performance Library contains a routine named DGBMV to multiply a banded matrix by a vector. By putting this routine into a properly constructed loop, Oracle Developer Studio Performance Library routines can be used to multiply a banded matrix by a matrix. The compiler will not parallelize this loop by default because the presence of subroutine calls in a loop inhibits parallelization. However, Oracle Developer Studio Performance Library routines are MT-safe, so you can use parallelization directives that instruct the compiler to parallelize this loop.

Compiler directives can also be used to parallelize a loop with a subroutine call that ordinarily would not be parallelizable. For example, it is ordinarily not possible to parallelize a loop containing a call to some of the linear system solvers, because some vendors have implemented those routines using code that is not MT-safe. Loops containing calls to the expert drivers of the linear system solvers (routines whose names end in SVX or SVXX) are usually not parallelizable with other implementations of LAPACK. Because the implementation of LAPACK in Oracle Developer Studio Performance Library enables parallelization of loops containing such calls, users of multiprocessor platforms can get additional performance by parallelizing these loops.

## C Interfaces

The Oracle Developer Studio Performance Library routines can be called from within a FORTRAN 77, Fortran 95, or C program. However, C programs must still use the FORTRAN 77 calling sequence.

Oracle Developer Studio Performance Library contains native C interfaces for each of the routines contained in LAPACK, BLAS, FFTPACK, VFFTPACK, SPARSE BLAS, and SPSOLVE. The Oracle Developer Studio Performance Library C interfaces have the following features:

- Function names have C names
- Function interfaces follow C conventions
- C interfaces do not contain redundant or unnecessary arguments for a $C$ function

The following example compares the standard LAPACK Fortran interface and the Oracle Developer Studio Performance Library C interfaces for the DGBCON routine.

```
CALL DGBCON (NORM, N, NSUB, NSUPER, DA, LDA, IPIVOT, DANORM,
    DRCOND, DWORK, IWORK2, INFO)
void dgbcon(char norm, int n, int nsub, int nsuper, double *da,
    int lda, int *ipivot, double danorm, double drcond,
```

```
int *info)
```

Note that the names of the arguments are the same and that arguments with the same name have the same base type. Scalar arguments that are used only as input values, such as NORM and N, are passed by value in the $C$ version. Arrays and scalars that will be used to return values are passed by reference.

The Oracle Developer Studio Performance Library C interfaces improve on CLAPACK, available on Netlib, which is an f2c translation of the standard libraries. For example, all of the CLAPACK routines are followed by a trailing underscore to maintain compatibility with Fortran compilers, which often postfix routine names in the object (.o) file with an underscore. The Oracle Developer Studio Performance Library C interfaces do not require a trailing underscore.

Oracle Developer Studio Performance Library C interfaces use the following conventions:

- Input-only scalars are passed by value rather than by reference. Complex and double complex arguments are not considered scalars because they are not implemented as a scalar type by C.
- Complex scalars can be passed as either structures or arrays of length 2.
- Types of arguments must match even after C does type conversion. For example, be careful when passing a single precision real value, because a C compiler can automatically promote the argument to double precision.
- Arrays are stored columnwise. For Fortran programmers, this is the natural order in which arrays are stored. For C programmers, this is the transpose of the order in which they usually work. In the documentation and man pages, references to rows refer to columns and vice versa.
- Array indices are based at one, in conformance with Fortran conventions, rather than being zero as in C.
For example, the Fortran interface to IDAMAX, which C programs access as idamax_, would return 1 to indicate the first element in a vector. The C interface to idamax, which C programs access as idamax, would also return a 1 to indicate the first element of a vector. This convention is observed in function return values, permutation vectors, and anywhere else that vector or array indices are used.

Note - Some Oracle Developer Studio Performance Library routines use malloc internally, so user codes that make calls to Oracle Developer Studio Performance Library and to sbrk might not work correctly.

The SPARC version of the Oracle Developer Studio Performance Library uses global integer registers $\% g 2, \% g 3$, and $\% g 4$ in 32-bit mode and $\% g 2$ through $\% g 5$ in 64-bit mode as scratch
registers. User code should not use these registers for temporary storage, and then call a Oracle Developer Studio Performance Library routine. The data will be overwritten when the Oracle Developer Studio Performance Library routine uses these registers.

## C Examples

Transforming user-written code sequences into calls to Oracle Developer Studio Performance Library routines increases application performance. The following code example adapted from LAPACK shows one example.

```
int i;
float a[n], b[n], largest;
largest = a[0];
for (i = 0; i < n; i++)
{
if (a[i] > largest)
    largest = a[i];
    if (b[i] > largest
    largest = b[i];
}
```

No Oracle Developer Studio Performance Library routine exactly replicates the functionality of this code example. However, the code can be accelerated by replacing it with several calls to the Oracle Developer Studio Performance Library routine isamax, as shown in the following code example.

```
int i, large_index;
float a[n], b[n], largest;
large_index = isamax (n, a, l) - 1;
largest = a[large_index];
large_index = isamax (n, b, l) - 1;
if (b[large_index] > largest)
    largest = b[large_index];
```

Compare the differences between calling the native C isamax routine in Oracle Developer Studio Performance Library, shown in the previous code example, with calling the isamax routine in CLAPACK, shown in the following code example.

```
/* 1. Declare scratch variable to allow 1 to be passed by reference */
int one = l;
/* 2. Append underscore to conform to FORTRAN naming system */
/* 3. Pass all arguments, even scalar input-only, by reference */
/* 4. Subtract one to convert from FORTRAN indexing conventions */
```

large_index = isamax_ (\&n, a, \&one) - l;
largest = a[large_index]; large_index = isamax_ (\&n, b, \&one) - l;
if (b[large_index] > largest)
largest $=\mathrm{b}[$ large_index];

## Optimizing Applications

This chapter describes how to use compiler and linking options to optimize applications for the following:

- Specific instruction-set architectures
- 32-bit and 64-bit enabled operating environments


## Comparison of 32-Bit and 64-Bit Environments

The following table shows a comparison of the 32-bit and 64-bit operating environments. These items are described in greater detail in the following sections.

TABLE 1 Comparison of 32-bit and 64-bit Operating Environments

|  | 32-bit (ILP 32) | 64-bit (LP64) |
| :--- | :--- | :--- |
| -xarch on SPARC <br> platforms | sparcvis, sparcvis2, <br> sparcfmaf | sparcvis, sparcvis2, sparcfmaf |
| -xarch on x86 platforms | generic, sse2 | sse2 |
| addressing | - m32 | $-m 64$ |
| Fortran Integers | INTEGER, INTEGER*4 | INTEGER*8 |
| C Integers | int | long |
| Floating-point | S/D/C/Z | S/D/C/Z |
| API | Names of routines | Names of routines with_64 suffix |

## Using the Oracle Developer Studio Performance Library

The Oracle Developer Studio Performance Library was compiled using the f95 compiler provided with this release. The Oracle Developer Studio Performance Library routines were compiled using -dalign, -xparallel.

## Linking Fortran Programs

When linking the program, use -dalign - library=sunperf and the same command line options that were used when compiling.

Oracle Developer Studio Performance Library is linked into an application with the - library switch rather than the $-l$ switch that is used to link in other libraries, as shown here.

```
my_system% f95 -dalign my_file.f -library=sunperf
```


## Linking C and C++ Programs

When linking your program, use - library=sunperf and the same command line options that were used when compiling. If you compile on a SPARC system, include the option xmemalign=8s as shown here. The -xmemalign=8s option is ignored on x86 and x64 platforms.
my_system\% cc -xmemalign=8s my_file.c -library=sunperf my_system\% CC -xmemalign=8s my_file.cpp -library=sunperf

If -dalign or -xmemalign=8s cannot be used for compilation, supply a trap 6 handler as described in "Enabling Trap 6 on SPARC Platforms" on page 17.

## About Compiling

Compile with the most appropriate -xarch option for best performance. At link time, use the same - xarch option that was used at compile time to select the version of the Oracle Developer Studio Performance Library optimized for a specific instruction-set architecture.

Note - The use of optimization options that are specific to the instruction set improves application performance on the selected instruction set architecture, but limits code portability.

For a detailed description of the different - xarch options, refer to the Oracle Developer Studio 12.6: Fortran User's Guide or the Oracle Developer Studio 12.6: C User's Guide.

The values for -xarch for SPARC and x86 instruction-set architectures are also listed in the man pages for fbe, cc, CC, and f95.

## Compiling Code for a 64-Bit Enabled Operating Environments

To compile code for a 64-bit enabled operating environment, use -m64 and convert all integer arguments to 64-bit arguments. 64-bit routines require the use of 64-bit integers.

Oracle Developer Studio Performance Library provides 32-bit and 64-bit interfaces. To use the 64-bit interfaces:

- Modify the Oracle Developer Studio Performance Library routine name. For C and Fortran 95 code, append _64 to the names of Oracle Developer Studio Performance Library routines (for example, rfftf_64 or CFFTB_64). For Fortran 95 code with the USE SUNPERF statement, the _64 suffix is not strictly required for specific interfaces, such as DGEMM. The _64 suffix is still required for the generic interfaces, such as GEMM.
- Promote integers to 64 bits. Double precision variables and the real and imaginary parts of double complex variables are already 64 bits. Only the integers are promoted to 64 bits.


## 64-Bit Integer Arguments

These additional 64-bit-integer interfaces are available only when linking with -m64. Codes compiled for 32-bit operating environments (-m32) cannot call the 64-bit-integer interfaces.

To call the 64-bit-integer interfaces directly, append the suffix _ 64 to the standard library name. For example, use daxpy_64() in place of daxpy ().

However, if calling the 64-bit integer interfaces indirectly, do not append _64 to the name of the Oracle Developer Studio Performance Library routine. Calls to the Performance Library routine will access a 32-bit wrapper that promotes the 32-bit integers to 64-bit integers, calls the 64-bit routine, and then demotes the 64-bit integers to 32-bit integers.

For best performance, call the routine directly by appending _64 to the routine name.
For C programs, use long instead of int arguments. The following code example shows calling the 64-bit integer interfaces directly.

```
#include <sunperf.h>
long n, incx, incy;
double alpha, *x, *y;
daxpy_64(n, alpha, x, incx, y, incy);
```

The following code example shows calling the 64-bit integer interfaces indirectly.

```
#include <sunperf.h>
int n, incx, incy;
double alpha, *x, *y;
daxpy (n, alpha, x, incx, y, incy);
```

For Fortran programs, use 64-bit integers for all integer arguments. The following methods can be used to convert integer arguments to 64-bits:

- To promote all integers that are declared without explicit byte sizes and literal integer constants from 32 bits to 64 bits, compile with -xtypemap=integer: 64 .
- To promote specific integer declarations, change INTEGER or INTEGER*4 to INTEGER*8.
- To promote integer literal constants, append $\_8$ to the constant.

Consider the following code example:

```
INTEGER*8 N
REAL*8 ALPHA, X(N), Y(N)
! _64 SUFFIX: N AND 1_8 ARE 64-BIT INTEGERS
CALL DAXPY_64(N,ALPHA,X,1_8,Y,1_8)
```

INTEGER*8 arguments cannot be used in a 32-bit environment. Routines in the 32-bit libraries cannot be called with 64-bit arguments. However, routines in the 64-bit libraries can be called with 32-bit arguments.

When passing constants in Fortran 95 code that have not been compiled with -xtypemap, append _ 8 to literal constants to effect the promotion. For example, when using Fortran 95, change CALL $\operatorname{DSCAL}(20,5.26 D 0, X, 1)$ to CALL $\operatorname{DSCAL}\left(20 \_8,5.26 D 0, X, 1 \_8\right)$. This example assumes USE SUNPERF is included in the code, because the _ 64 has not been appended to the routine name.

The following code example shows calling CAXPY from Fortran 95 using 32-bit arguments.

```
PROGRAM TEST
COMPLEX ALPHA
INTEGER,PARAMETER :: INCX=1, INCY=1, N=10
COMPLEX X(N), Y(N)
CALL CAXPY(N, ALPHA, X, INCX, Y, INCY)
```

The following code example shows calling CAXPY from Fortran 95 (without the USE SUNPERF statement) using 64-bit arguments.

```
PROGRAM TEST
COMPLEX ALPHA
INTEGER*8, PARAMETER :: INCX=1, INCY=1, N=10
COMPLEX X(N), Y(N)
```

CALL CAXPY_64(N, ALPHA, X, INCX, Y, INCY)
When using 64-bit arguments, the _64 must be appended to the routine name if the USE SUNPERF statement is not used.

The following Fortran 95 code example shows calling CAXPY using 64-bit arguments.

```
PROGRAM TEST
USE SUNPERF
COMPLEX ALPHA
INTEGER*8, PARAMETER :: INCX=1, INCY=1, N=10
COMPLEX X(N), Y(N)
CALL CAXPY(N, ALPHA, X, INCX, Y, INCY)
```

In C routines, the size of long is 32 bits when compiling with -m32and 64 bits when compiling with -m64. The following code example shows calling the dgbcon routine using 32-bit arguments.

```
void dgbcon(char norm, int n, int nsub, int nsuper, double *da,
    int lda, int *ipivot, double danorm, double drcond,
    int *info)
```

The following code example shows calling the dgbcon routine using 64-bit arguments.

```
void dgbcon_64 (char norm, long n, long nsub, long nsuper,
    double *da, long lda, long *ipivot, double danorm,
    double *drcond, long *info)
```


## Parallel Processing

This chapter describes using the Oracle Developer Studio Performance Library in multiprocessor environments with shared memory parallelism.

## Run-Time Issues

At run time, if running with compiler parallelization, the Oracle Developer Studio Performance Library uses the same pool of threads that the compiler does. The per-thread stack size must be set to at least 8 Mbytes on all platforms. This is done with the STACKSIZE or the OMP_STACKSIZE environment variable (units in Kbytes). You cannot use both. If you use the two environment variables simultaneously and the two values are different, the program stops with an error message.

To set the per-thread stack size to 8 Mbytes :
my_host\% setenv STACKSIZE 8192
Setting the STACKSIZE environment variable is not required for programs running with POSIX or Oracle Solaris threads. In this case, user-created threads that call Performance Library routines must have a stack size of at least 8 Mbytes. Failure to supply an adequate stack size for the Performance Library routines might result in stack overflow problems. Symptoms of stack overflow problems include runtime failures that could be difficult to diagnose. For more information on setting the stack size of user-created threads, see the pthread_create(3C), pthread_attr_init(3C), and pthread_attr_setstacksize(3C) man pages for POSIX threads or the thr_create(3C) for Oracle Solaris threads.

Tip - If you are having issues diagnosing a core dump, try increasing the stack size above the minimum.

## Degree of Parallelism

Selected routines in the Oracle Developer Studio Performance Library are parallelized using compiler directives, library routines, and environment variables from the OpenMP Fortran Application Program Interface. The number of threads these routines use in parallel is controlled by the environment variable OMP_NUM_THREADS. You can also set the environment variable PARALLEL, but if you set both they must have the same value or a fatal error will occur upon execution. Both environment variables can be overridden by calling the Oracle Developer Studio Performance Library routine USE_THREADS or the OpenMP routine OMP_SET_NUM_THREADS in the user code.

A user code can be parallelized by doing the following:

- Set environment variable OMP_NUM_THREADS to a value greater than 1
- Use compiler parallel directives such as those from the OpenMP API

Use appropriate compiler flags: -xopenmp=parallel or -xautopar
The Oracle Developer Studio Performance Library routines execute in parallel if the following conditions are met:

- OMP_NUM_THREADS is set to a value greater than 1
- The routines are not being called from a parallel region

The Oracle Developer Studio Performance Library employs OpenMP directives in its parallelization and does not support nested parallelism. If the user code is parallelized as stated above and calls a Oracle Developer Studio Performance Library routine, the routine executes in serial if it detects that it is being called from a parallel region. Otherwise, the routine executes in parallel.

POSIX or Oracle Solaris threads can also be created to execute in parallel selected regions in the user code. When a Performance Library routine is called under this parallel model, the routine cannot detect that it is being called from a parallel region. Therefore, the environment variable OMP_NUM_THREADS must be set to 1 or unset, or a call to USE_THREADS(3P) must be made in appropriate places in the user code. Otherwise, nested parallelism with undefined results will occur.

For example, if the program containing the following code segment is linked with xopenmp=parallel and OMP_NUM_THREADS is set to 4, the loop will execute in parallel, and there will be four instances of DGEMM running concurrently. However, each DGEMM instance will run in serial since only one level of parallelization is supported.

```
!$OMP PARALLEL
    DO I = 1, N
            CALL DGEMM(...)
```

```
END DO
!\$OMP END PARALLEL
```

In the following code example, if the program is not linked with -xautopar, the loop will not be parallelized, but each instance of DGEMM will be executed by four threads.

```
DO I = 1, N
    CALL DGEMM(...)
END DO
```

If the program containing the following code segment is linked with -xopenmp=parallel and if OMP_NUM_THREADS is set to a value greater than 1, the region shown will be executed by a single thread. However, each DGEMM call will be executed by OMP_NUM_THREADS threads.

```
!$OMP SINGLE
    DO I = 1, N
        CALL DGEMM(...)
    END DO
!$OMP END SINGLE
```

In the following code example, there will be at most two-way parallelism, regardless of the number of OpenMP threads available for execution. Only one level of parallelism exists, which are the two sections. Further parallelism within a DGEMM call is suppressed.

```
!$OMP PARALLEL SECTIONS
!$OMP SECTION
    DO I = 1, N / 2
        CALL DGEMM(...)
    END DO
!$OMP SECTION
    DO I = N / 2 + 1, N
        CALL DGEMM(...)
    END DO
!$OMP END PARALLEL SECTIONS
```


## Synchronization Mechanisms

One characteristic of the POSIX/Oracle Solaris threading model is that bound threads of a running application relinquish the CPUs when they are idle, thus providing good throughput and resource usage in a shared (over-subscribed) environment. By default, bound threads in a compiler-parallelized code spin-wait when they are idle, which can result in suboptimal throughput when there are other applications in the system competing for CPU resource. In this case, environment variable SUNW_MP_THR_IDLE can be used to control the behavior of a thread after it finishes its share of a parallel job:
my_host\% setenv SUNW_MP_THR_IDLE value
Here, value can either be spin or sleep $n$ s or sleep $n \mathrm{~ms}$, and spin is the default.
sleep puts the thread to sleep after spin-waiting $n$ units. The wait unit can be seconds ( $s$, the default unit) or milliseconds (ms). sleep with no arguments puts the thread to sleep immediately after completing a parallel task. If SUNW_MP_THR_IDLE contains an illegal value or is not set, spin is used as the default.

The following settings would cause threads to spin-wait (default behavior), spin for 2 seconds before sleeping, or spin for 100 milliseconds before sleeping, respectively. Using Oracle Developer Studio Performance Library routines does not change the spin-wait behavior of the code.

```
% setenv SUNW_MP_THR_IDLE spin
% setenv SUNW_MP_THR_IDLE 2s
% setenv SUNW_MP_THR_IDLE 100ms
```


## Parallel Processing Examples

This section demonstrates how to use the OMP_NUM_THREADS environment variable along with compile and link options to create code that executes serially and in parallel.

To create a serial application:

- Call one or more Oracle Developer Studio Performance Library routines
- Link with - library=sunperf, placing the flag at the end of the command line. Do not compile or link with -xopenmp=parallel, or -xautopar
- Unset OMP_NUM_THREADS environment variable or set it to 1

The following examples show how to compile and link with the shared Oracle Developer Studio Performance library libsunperf.so.

```
my_host\% cc -xmemalign=8s -xarch=native any.c -library=sunperf
my_host\% f95 -dalign -xarch=native any.f95 -library=sunperf
```

To create a parallel application that executes on multiple processors:

- Call one or more Oracle Developer Studio Performance Library routines
- Use the same parallelization option (-xopenmp=parallel or -xautopar) in the compile and link commands
- Link with -library=sunperf, placing the flag at the end of the command line
- Set OMP_NUM_THREADS to the number of available processors before running the executable For example, to use 24 processors, type the following commands:

```
my_host% f95 -dalign -xarch=native my_app.f -library=sunperf
my_host% setenv OMP_NUM_THREADS 24
my_host% ./a.out
```

The previous example enables Oracle Developer Studio Performance Library routines to run in parallel, but no part of the user code my_app. f will run in parallel. For the compiler to attempt to parallelize my_app.f, either -xopenmp=parallel or -xautopar is required on the compile line:
my_host\% f95 -dalign -xopenmp=parallel my_app.f -library=sunperf my_host\% setenv OMP_NUM_THREADS 24
my_host\% ./a.out

## Working With Matrices

Most matrices can be stored in ways that save both storage space and computation time. Oracle Developer Studio Performance Library uses the following storage schemes:

- Banded storage
- Packed storage

The Oracle Developer Studio Performance Library processes matrices that are in one of four forms:

- General
- Triangular
- Symmetric
- Tridiagonal

Storage schemes and matrix types are described in the following sections.

## Matrix Storage Schemes

Some Oracle Developer Studio Performance Library routines that work with arrays stored normally have corresponding routines that take advantage of these special storage forms. For example, DGBMV will form the product of a general matrix in banded storage and a vector, and DTPMV will form the product of a triangular matrix in packed storage and a vector.

## Banded Storage

A banded matrix is stored so the $j$ th column of the matrix corresponds to the $j$ th column of the Fortran array.

The following code copies a banded general matrix in a general array into banded storage mode.

```
C Copy the matrix A from the array AG to the array AB. The
C matrix is stored in general storage mode in AG and it will
C be stored in banded storage mode in AB. The code to copy
C from general to banded storage mode is taken from the
C comment block in the original DGBFA by Cleve Moler.
C
NSUB = 1
NSUPER = 2
NDIAG = NSUB + 1 + NSUPER
DO ICOL = 1, N
    I1 = MAX0 (1, ICOL - NSUPER)
    I2 = MIN0 (N, ICOL + NSUB)
    DO IROW = I1, I2
        IROWB = IROW - ICOL + NDIAG
        AB(IROWB,ICOL) = AG(IROW,ICOL)
    END DO
END DO
```

This method of storing banded matrices is compatible with the storage method used by LAPACK and BLAS.

## Packed Storage

A packed vector is an alternative representation for a triangular, symmetric, or Hermitian matrix. An array is packed into a vector by storing the elements sequentially column by column into the vector. Space for the diagonal elements is always reserved, even if the values of the diagonal elements are known, such as in a unit diagonal matrix.

An upper triangular matrix or a symmetric matrix whose upper triangle is stored in general storage in the array A, can be transferred to packed storage in the array AP as shown below. This code comes from the comment block of the LAPACK routine DTPTRI.

```
JC = 1
DO J = 1, N
    DO I = 1, J
        AP(JC+I-1) = A(I,J)
    END DO
    JC = JC + J
END DO
```

Similarly, a lower triangular matrix or a symmetric matrix whose lower triangle is stored in general storage in the array A, can be transferred to packed storage in the array AP as shown below:

$$
J C=1
$$

```
DO J = 1, N
    DO I = J, N
        AP(JC+I-1) = A(I,J)
    END DO
    JC = JC + N - J + 1
END DO
```


## Rectangular Full Packed Format

Rectangular Full Packed (RFP) matrices is a data format for storing triangular and symmetric matrices. It combines the standard packed format arrays fully utilized storage with high performance using level 3 BLAS. For information, see Rectangular Full Packed Format for LAPACK Algorithms Timings on Several Computers (http://citeseerx.ist.psu.edu/ viewdoc/download?doi=10.1.1.456.7563\&rep=rep1\&type=pdf) and "Further Details" section of the man pages for the routines that use Rectangular Full Packed Format.

## Matrix Types

The general matrix is the most common type, and most operations in the Oracle Developer Studio Performance Library operate on the general matrix. In many cases, there are routines that will work with the other types of matrices. For example, DGEMM computes the product of two general matrices, and DTRMM computes the product of a triangular matrix and a general matrix.

## General Matrices

The storage of a general matrix is such that there is a one-to-one correspondence between the elements of the matrix and the elements of the array. Element Aij of matrix A is stored in element $\mathrm{A}(I, J)$ of the corresponding array A . The general matrix has no special storage scheme since each of its elements is stored explicitly. In contrast, only the nonzero upper-diagonal, diagonal, and lower-diagonal elements of a general band matrix are stored. The following example shows how a general band matrix is stored in a two-dimensional array. Array locations marked with x are not accessed.

| General Band Matrix |
| :--- |
| $\left[\begin{array}{ccccc}a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55}\end{array}\right] \quad\left[\begin{array}{ccccc}x & x & a_{13} & a_{24} & a_{35} \\ x & a_{12} & a_{23} & a_{34} & a_{45} \\ a_{11} & a_{22} & a_{33} & a_{44} & a_{55} \\ a_{21} & a_{32} & a_{43} & a_{54} & x\end{array}\right]$ |

## Triangular Matrices

Two storage schemes exist for a triangular matrix. In the unpacked scheme where the matrix is stored in a two-dimensional array, there is a one-to-one correspondence between all elements of the matrix and the elements of the array, but zero entries in the matrix are neither set nor accessed in the array (denoted by $x$ ). In the packed storage scheme, nonzero elements of the matrix are packed by column in a one-dimensional array.

A triangular matrix can be stored using packed storage.
$\left.\begin{array}{lll}\hline \text { Triangular Band Matrix } & \begin{array}{l}\text { Triangular Matrix in } \\ \text { Unpacked Storage }\end{array} & \begin{array}{l}a_{11} \\ a_{21}\end{array} a_{22} \\ a_{31} & a_{32} & a_{33}\end{array}\right]\left[\begin{array}{lll}a_{11} & x & x \\ a_{21} & a_{22} & x \\ a_{31} & a_{32} & a_{33}\end{array}\right]\left[\begin{array}{l}\text { Triangular Matrix in Packed } \\ \text { Storage }\end{array}\right]\left[\begin{array}{l}a_{11} \\ a_{21} \\ a_{31} \\ a_{22} \\ a_{32} \\ a_{33}\end{array}\right]$

A triangular band matrix can be stored in packed storage using a two-dimensional array as shown below. Elements marked with x are not accessed.

| Triangular Band Matrix | Triangular Band Matrix in <br> Packed Storage |
| :---: | :--- |
| $\left[\begin{array}{ccc}a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ 0 & a_{32} & a_{33}\end{array}\right]$ | $\left[\begin{array}{lll}a_{11} & a_{22} & a_{33} \\ a_{21} & a_{32} & x\end{array}\right]$ |

## Symmetric Matrices

A real symmetric or complex Hermitian matrix is similar to a triangular matrix in that only elements in its upper or lower triangle are explicitly stored in the corresponding elements of a two-dimensional array. The remaining elements of the array (denoted by $x$ below) are neither set nor accessed. The active upper or lower triangle can also be packed by column into a onedimensional array.
$\left.\begin{array}{ll|l|l}\hline \text { Symmetric Matrix } & & \begin{array}{l}\text { Symmetric Matrix in } \\ \text { Unpacked Storage }\end{array} & \begin{array}{l}\text { Symmetric Matrix in } \\ a_{11}\end{array} a_{12} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{array}\right]\left[\begin{array}{lll}a_{11} & x & x \\ a_{21} & a_{22} & x \\ a_{31} & a_{32} & a_{33}\end{array}\right]\left[\begin{array}{l}a_{11} \\ a_{21} \\ a_{31} \\ a_{22} \\ a_{32} \\ a_{33}\end{array}\right]$

## Tridiagonal Matrices

A tridiagonal matrix has nonzero elements only on the main diagonal, the first superdiagonal, and the first subdiagonal. It is stored using three one-dimensional arrays.

| Tridiagonal Matrix |  |
| :--- | :--- |
| $\left[\begin{array}{ccc}a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \\ 0 & 0 & a_{43}\end{array}\right.$ |  |
| Matrix |  |\(\quad\left[\begin{array}{l}a_{21} <br>

a_{32} <br>
a_{43}\end{array}\right]\left[$$
\begin{array}{l}a_{11} \\
a_{22} \\
a_{33} \\
a_{44}\end{array}
$$\right]\left[$$
\begin{array}{l}a_{12} \\
a_{23} \\
a_{34}\end{array}
$$\right]\)

CHAPTER 6

## Sparse Computation

The Oracle Developer Studio Performance Library has two software packages, SPSOLVE and SuperLU, that can be used to factor and solve sparse linear systems of equations.

SPSOLVE is a collection of routines that solve symmetric, structurally symmetric, and unsymmetric coefficient matrices using one of several ordering methods, including a userspecified ordering. In previous releases, SPSOLVE was referred to as the sparse solver package. It is written mainly in Fortran and contains interfaces for FORTRAN 77 only. Fortran 95 and C interfaces are not currently provided. To use SPSOLVE routines from Fortran 95, use the FORTRAN 77 interfaces. To call SPSOLVE from C, append an underscore to the routine name (dgssin_(), dgssor_(), and so on), pass arguments by reference, and use one-based array indexing. See "Unsymmetric Sparse Matrices" on page 49 for an example of one-based and zero-based array indexing.

The SuperLU package in the Oracle Developer Studio Performance Library is the sequential version (version 3.0) of the public domain application that solves general unsymmetric sparse systems. While it is sequential, SuperLU does make use of several level 2 and level 3 BLAS routines that are parallelized. For detailed documentation of SuperLU algorithm, routines and data structures, see items 5, 6, 7 in "References for Sparse BLAS and Solver" on page 75. SuperLU is written in C, which requires array indexing to be zero-based regardless of whether SuperLU routines are being called from Fortran-based SPSOLVE or a C driver program. See "SuperLU Interface" on page 64 for more detail and examples.

## Sparse Matrices

Sparse matrices are usually represented in formats that minimize storage requirements. By taking advantage of the sparsity and not storing zeros, considerable storage space can be saved. The storage format used by SPSOLVE and SuperLU is the compressed sparse column (CSC) format, also called the Harwell-Boeing format.

The CSC format represents a sparse matrix with two integer arrays and one floating point array. The integer arrays (colptr and rowind) specify the location of the nonzeros of the sparse matrix, and the floating point array (values) is used for the nonzero values.

The column pointer (colptr) array consists of $\mathrm{n}+1$ elements where colptr(i) points to the beginning of the $\mathrm{i}^{\text {th }}$ column, and colptr( $\mathrm{i}+1$ )-1 points to the end of the $\mathrm{i}^{\text {th }}$ column. The row indices (rowind) array contains the row indices of the nonzero values. The values arrays contains the corresponding nonzero numerical values.

The following matrix data formats exist for a sparse matrix of neqns equations and nnz nonzeros:

- Symmetric
- Structurally symmetric
- Unsymmetric

Currently, SuperLU only supports unsymmetric matrices. The most efficient data representation often depends on the specific problem. The following sections show examples of sparse matrix data formats.

## Symmetric Sparse Matrices

A symmetric sparse matrix is a matrix where $\mathrm{a}(i, j)=\mathrm{a}(j, i)$ for all $i$ and $j$. Because of this symmetry, only the lower triangular values need to be passed to the solver routines. The upper triangle can be determined from the lower triangle.

An example of a symmetric matrix is shown below. This example is derived from A. George and J. W-H. Liu. "Computer Solution of Large Sparse Positive Definite Systems."

$$
A=\left[\begin{array}{ccccc}
4.0 & 1.0 & 2.0 & 0.5 & 2.0 \\
1.0 & 0.5 & 0.0 & 0.0 & 0.0 \\
2.0 & 0.0 & 3.0 & 0.0 & 0.0 \\
0.5 & 0.0 & 0.0 & 0.625 & 0.0 \\
2.0 & 0.0 & 0.0 & 0.0 & 16.0
\end{array}\right]
$$

To represent $A$ in CSC format:

- colptr: $1,6,7,8,9,10$
- rowind: $1,2,3,4,5,2,3,4,5$
- values: $4.0,1.0,2.0,0.5,2.0,0.5,3.0,0.625,16.0$


## Structurally Symmetric Sparse Matrices

A structurally symmetric sparse matrix has nonzero values with the property that if a $(i, j) \neq 0$, then $\mathrm{a}(j, i) \neq 0$ for all $i$ and $j$. When solving a structurally symmetric system, the entire matrix must be passed to the solver routines.

An example of a structurally symmetric matrix is shown below.

$$
A=\left[\begin{array}{llll}
1.0 & 3.0 & 0.0 & 0.0 \\
2.0 & 4.0 & 0.0 & 7.0 \\
0.0 & 0.0 & 6.0 & 0.0 \\
0.0 & 5.0 & 0.0 & 8.0
\end{array}\right]
$$

To represent $A$ in CSC format:

- colptr: 1, 3, 6, 7, 9
- rowind: $1,2,1,2,4,3,2,4$
- values: $1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0$


## Unsymmetric Sparse Matrices

An unsymmetric sparse matrix does not have $\mathrm{a}(i, j)=\mathrm{a}(j, i)$ for all $i$ and $j$. The structure of the matrix does not have an apparent pattern. When solving an unsymmetric system, the entire matrix must be passed to the solver routines. An example of an unsymmetric matrix is shown below.

$$
A=\left[\begin{array}{ccccc}
1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
2.0 & 6.0 & 0.0 & 0.0 & 9.0 \\
3.0 & 0.0 & 7.0 & 0.0 & 0.0 \\
4.0 & 0.0 & 0.0 & 8.0 & 0.0 \\
5.0 & 0.0 & 0.0 & 0.0 & 10.0
\end{array}\right]
$$

To represent $A$ in CSC format:

- One-based indexing:
- colptr: $1,6,7,8,9,11$
- rowind: $1,2,3,4,5,2,3,4,2,5$
- values: $1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0,9.0,10.0$
- Zero-based indexing:
- colptr: $0,5,6,7,8,10$
- rowind: $0,1,2,3,4,1,2,3,1,4$
- values: $1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0,9.0,10.0$


## Sparse BLAS

The Oracle Developer Studio Performance Library Sparse BLAS package is based on the following two packages:

- Netlib Sparse BLAS package, by Dodson, Grimes, and Lewis consists of sparse extensions to the Basic Linear Algebra Subroutines that operate on sparse vectors.
- NIST (National Institute of Standards and Technology) Fortran Sparse BLAS Library consists of routines that perform matrix products and solution of triangular systems for sparse matrices in a variety of storage formats.

Refer to the following sources for additional Sparse BLAS information.

- For information on the Sparse BLAS routines, refer to the section 3P man pages for the individual routines.
- For more information on the Netlib Sparse BLAS package refer to http://www. netlib. org/sparse-blas/index.html.
- For more information on the NIST Fortran Sparse BLAS routines, refer to http://math. nist.gov/spblas/.

The Netlib Sparse BLAS and NIST Fortran Sparse BLAS Library routines each use their own naming conventions, as described in the following sections.

## Netlib Sparse BLAS

Each Netlib Sparse BLAS routine has a name of the form Prefix-Root-Suffix:

- Prefix represents the data type.
- Root represents the operation.
- Suffix represents whether or not the routine is a direct extension of an existing dense BLAS routine.

The following table lists the naming conventions for the Netlib Sparse BLAS vector routines.

TABLE 2 Netlib Sparse BLAS Naming Conventions

| Operation | Root of Name | Prefix and Suffix |
| :--- | :--- | :--- |
| Dot product | -DOT- | S-I D-I C-UI Z-UI C-CI Z-CI |
| Scalar times a vector added <br> to a vector | -AXPY- | S-I D-I C-I Z-I |
| Apply Givens rotation | -ROT- | S-I D-I |
| Gather x into y | -GTHR- | S-D-C- Z- S-Z D-Z C-Z Z-Z |
| Scatter $x$ into y | -SCTR- | S-D-C- Z- |

The prefix can be one of the following data types:

- S: SINGLE
- D: DOUBLE
- C: COMPLEX
- Z: COMPLEX*16 or DOUBLE COMPLEX

The I, CI, and UI suffixes denote sparse BLAS routines that are direct extensions to dense BLAS routines.

## NIST Fortran Sparse BLAS

Each NIST Fortran Sparse BLAS routine has a six-character name of the form XYYYZZ where:

- $X$ represents the data type.
- YYY represents the sparse storage format.
- $Z Z$ represents the operation.

The following table shows the possible values for $X, Y Y Y$, and $Z Z$.

TABLE 3 NIST Fortran Sparse BLAS Routine Naming Conventions

| Variables for a Routine Name | Acceptable Values and Meaning |
| :--- | :--- |
| $X$ - Specifies the data type using one character | S: single precision |
|  | D: double precision |
|  | C: complex |
|  | Z: double complex |


| Variables for a Routine Name | Acceptable Values and Meaning |
| :--- | :--- |
| YYY - Specifies the sparse storage format using three <br> characters | Single entry formats: |
|  | CSC: compressed sparse column |
| COO: coordinate |  |
| CSR: compressed sparse row |  |
| DIA: diagonal |  |
| ELL: ellpack |  |
|  | JAD: jagged diagonal |
|  | SKY: skyline |
|  | Block entry formats: |
|  | BCO: block coordinate |
|  | BSC: block compressed sparse column |
| BSR: block compressed sparse row |  |
|  | BDI: block diagonal |
|  | BEL: block ellpack |
|  | VBR: block compressed sparse row |
| MM: matrix-matrix product |  |
|  | SM: solution of triangular system (supported for all |
| formats except COO) |  |
|  | RP: right permutation (for JAD format only) |

## SPSOLVE Interface

SPSOLVE computes the solution of a sparse system through a sequence of steps: Initialization, ordering to reduce fill-in, symbolic factorization, numeric factorization, and triangular solve. A user code can call individual routines or make use of a one-call interface to perform these steps.

## SPSOLVE Routines

Listed in the table below are user-accessible routines in SPSOLVE and their purposes.

TABLE 4 SPSOLVE Sparse Solver Routines

| Routine Name | Description |
| :--- | :--- |
| DGSSFS () | One-call interface to sparse solver |
| DGSSIN ( ) | Sparse solver initialization |
| DGSSOR() | Fill reducing ordering and symbolic factorization |
| DGSSUO() | Sets user-specified ordering permutation and performs <br> symbolic factorization (called in place of DGSSOR) |


| Routine Name | Description |
| :--- | :--- |
| DGSSFA ( ) | Matrix value input and numeric factorization |
| DGSSSL ( $)$ | Triangular solve |
| $\operatorname{DGSSRP}()$ | Returns permutation used by solver |
| $\operatorname{DGSSCO}()$ | Returns condition number estimate of coefficient matrix |
| $\operatorname{DGSSDA}()$ | De-allocates sparse solver |
| DGSSPS () | Prints solver statistics |

Matrices with the same structure but with different numerical values can be solved by calling SPSOLVE routines in the following order shown:

```
call dgssin() ! initialization, input coefficient matrix structure
call dgssor() ! fill-reducing ordering, symbolic factorization
    ! (or call dgssuo() to specify a user ordering,
    ! and perform symbolic factorization)
do m = 1, number_of_structurally_identical_matrices
    call dgssfa() ! input coefficient matrix values, numeric !
    factorization
    do r = 1, number_of_right_hand_sides
        call dgsssl() ! triangular solve
    enddo
enddo
```

The one-call interface is not as flexible as the regular interface, but it covers the most common case of factoring a single matrix and solving some number of right-hand sides. Additional calls to dgsssl () are used to solve for additional right-hand sides, as shown in the following example.

```
call dgssfs() ! initialization, input coefficient matrix structure
    ! fill-reducing ordering, symbolic factorization
    ! input coefficient matrix values, numeric factorization
    ! triangular solve
do r = 1, number_of_right_hand_sides
    call dgsssl() ! triangular solve
enddo
```


## SPSOLVE Routine Calling Order

To use SPSOLVE, you must call its routines in the following order shown:

1. One-Call Interface: For solving single matrix
a. DGSSFS () - Initialize, order, factor, solve
b. DGSSSL() - Additional solves (optional): repeat DGSSSL() as needed
c. DGSSDA () - Deallocate working storage
2. Regular Interface: For solving multiple matrices with the same structure
a. DGSSIN() - Initialize
b. DGSSOR() or DGSSUO() - Order and symbolically factor
c. DGSSFA() - Factor
d. DGSSSL () - Solve: repeat DGSSFA () or DGSSSL () as needed
e. DGSSDA() - Deallocate working storage

## SPSOLVE Examples

The following examples show solving a symmetric system using the one-call interface, and solving a symmetric system using the regular interface.

In Example 1, "Solving a Symmetric System-One-Call Interface," on page 54, the one-call interface is used to solve a symmetric system, and in Example 2, "Solving a Symmetric System - Regular Interface," on page 56, individual routines are called to solve a symmetric system.

Example 5, "Calling SPSOLVE Routines from C," on page 63 shows how the Fortran SPSOLVE interface can be called from a C program. For more information on how to call Fortran routines from C programs, see the Oracle Solaris Studio 12.4: Fortran Programming Guide.

EXAMPLE 1 Solving a Symmetric System-One-Call Interface
my_system\% cat example_1call.f
program example_1call
c
c This program is an example driver that calls the sparse solver.
c It factors and solves a symmetric system, by calling the c one-call interface.
c
implicit none
integer neqns, ier, msglvl, outunt, ldrhs, nrhs
character mtxtyp*2, pivot*1, ordmthd*3
double precision handle(150)
integer colstr(6), rowind(9)
double precision values(9), rhs(5), xexpct(5)
integer i

```
c
c Sparse matrix structure and value arrays. From George and Liu,
c page 3.
c Ax = b, (solve for x) where:
C
c}\begin{array}{llllllll}{4.0}&{1.0}&{2.0}&{0.5}&{2.0}&{2.0}&{7.0}
c}\begin{array}{lllllll}{1.0}&{0.5}&{0.0}&{0.0}&{0.0}&{2.0}&{3.0}
c A = 2.0 0.0 3.0 0.0 0.0 x = 1.0 b = 7.0
c 0.5 0.0
c 
C
        data colstr / 1, 6, 7, 8, 9, 10 /
        data rowind / 1, 2, 3, 4, 5, 2, 3, 4, 5 /
        data values / 4.0d0, 1.0d0, 2.0d0, 0.5d0, 2.0d0, 0.5d0, 3.0d0,
        & 0.625d0, 16.0d0 /
        data rhs / 7.0d0, 3.0d0, 7.0d0, -4.0d0, -4.0d0 /
        data xexpct / 2.0d0, 2.0d0, 1.0d0, -8.0d0, -0.5d0 /
C
c set calling parameters
c
        mtxtyp= 'ss'
        pivot = 'n'
        neqns = 5
        nrhs = 1
        ldrhs = 5
        outunt = 6
        msglvl = 0
        ordmthd = 'mmd'
C
c call single call interface
C
        call dgssfs ( mtxtyp, pivot, neqns , colstr, rowind,
        & values, nrhs , rhs, ldrhs , ordmthd,
        & outunt, msglvl, handle, ier )
        if ( ier .ne. 0 ) goto 110
C
c deallocate sparse solver storage
c
        call dgssda ( handle, ier )
        if ( ier .ne. 0 ) goto 110
c
c print values of sol
c
    write(6,200) 'i', 'rhs(i)', 'expected rhs(i)', 'error'
    do i = 1, neqns
        write(6,300) i, rhs(i), xexpct(i), (rhs(i)-xexpct(i))
    enddo
```

```
            stop
        110 continue
c
c call to sparse solver returns an error
c
    write ( 6 , 400 )
        & ' example: FAILED sparse solver error number = ', ier
        stop
        200 format(a5,3a20)
        300 format(i5,3d20.12) ! i/sol/xexpct values
        400 format(a60,i20) ! fail message, sparse solver error number
        end
my_system% f95 -dalign example_1call.f -library=sunperf
my_system% a.out
    i rhs(i) expected rhs(i) error
    0.200000000000D+01 0.200000000000D+01 -0.528466159722D-13
    0.200000000000D+01 0.200000000000D+01 0.105249142734D-12
    0.100000000000D+01 0.100000000000D+01 0.350830475782D-13
    -0.800000000000D+01 -0.800000000000D+01 0.426325641456D-13
    5-0.500000000000D+00 -0.500000000000D+00 0.660582699652D-14
```


## EXAMPLE 2 Solving a Symmetric System - Regular Interface

```
my_system% cat example_ss.f
```

        program example_ss
    c
c This program is an example driver that calls the sparse solver.
c It factors and solves a symmetric system.
implicit none
integer neqns, ier, msglvl, outunt, ldrhs, nrhs
character mtxtyp*2, pivot*1, ordmthd*3
double precision handle(150)
integer colstr(6), rowind(9)
double precision values(9), rhs(5), xexpct(5)
integer i
c
c Sparse matrix structure and value arrays. From George and Liu,
c page 3 .
c $\quad A x=b$, (solve for $x$ ) where:
c

```
C 4.0
c}\begin{array}{lllllll}{1.0}&{0.5}&{0.0}&{0.0}&{0.0}&{2.0}&{3.0}
c A = 2.0 0.0 3.0 0.0 0.0 x = 1.0 b = 7.0
c 0.5 0.0
c llllllll
c
        data colstr / 1, 6, 7, 8, 9, 10 /
        data rowind / 1, 2, 3, 4, 5, 2, 3, 4, 5 /
        data values / 4.0d0, 1.0d0, 2.0d0, 0.5d0, 2.0d0, 0.5d0,
        & 3.0d0, 0.625d0, 16.0d0 /
        data rhs / 7.0d0, 3.0d0, 7.0d0, -4.0d0, -4.0d0 /
        data xexpct / 2.0d0, 2.0d0, 1.0d0, -8.0d0, -0.5d0 /
C
c initialize solver
C
        mtxtyp= 'ss'
        pivot = 'n'
        neqns = 5
        outunt = 6
        msglvl = 0
c
c call regular interface
C
        call dgssin ( mtxtyp, pivot, neqns , colstr, rowind,
        & outunt, msglvl, handle, ier )
        if ( ier .ne. 0 ) goto 110
C
c ordering and symbolic factorization
c
        ordmthd = 'mmd'
        call dgssor ( ordmthd, handle, ier )
        if ( ier .ne. 0 ) goto 110
C
c numeric factorization
c
        call dgssfa ( neqns, colstr, rowind, values, handle, ier )
        if ( ier .ne. 0 ) goto 110
C
C solution
C
        nrhs = 1
        ldrhs = 5
        call dgsssl ( nrhs, rhs, ldrhs, handle, ier )
        if ( ier .ne. 0 ) goto 110
C
c deallocate sparse solver storage
C
```

```
            call dgssda ( handle, ier )
            if ( ier .ne. 0 ) goto 110
C
c print values of sol
c
    write(6,200) 'i', 'rhs(i)', 'expected rhs(i)', 'error'
        do i = 1, neqns
            write(6,300) i, rhs(i), xexpct(i), (rhs(i)-xexpct(i))
        enddo
        stop
        1 1 0 \text { continue}
C
c call to sparse solver returns an error
c
    write ( 6 , 400 )
        & ' example: FAILED sparse solver error number = ', ier
        stop
        200 format(a5,3a20)
    300 format(i5,3d20.12) ! i/sol/xexpct values
    400 format(a60,i20) ! fail message, sparse solver error number
        end
my_system% f95 -dalign example_ss.f -library=sunperf
my_system% a.out
    i rhs(i) expected rhs(i) error
    1 0.200000000000D+01 0.200000000000D+01 -0.528466159722D-13
    2 0.200000000000D+01 0.200000000000D+01 0.105249142734D-12
    3 0.100000000000D+01 0.100000000000D+01 0.350830475782D-13
    4-0.800000000000DD+01 -0.8000000000000D+01 0.426325641456D-13
    5-0.5000000000000D+00 -0.500000000000D+00 0.660582699652D-14
```

EXAMPLE 3 Solving a Structurally Symmetric System With Unsymmetric Values - Regular Interface

```
my_system% cat example_su.f
    program example_su
C
c This program is an example driver that calls the sparse solver.
c It factors and solves a structurally symmetric system
c (w/unsymmetric values).
C
    implicit none
```

```
integer neqns, ier, msglvl, outunt, ldrhs, nrhs
character mtxtyp*2, pivot*1, ordmthd*3
double precision handle(150)
integer colstr(5), rowind(8)
double precision values(8), rhs(4), xexpct(4)
integer i
C
c Sparse matrix structure and value arrays. Coefficient matrix
c has a symmetric structure and unsymmetric values.
c Ax = b, (solve for }x\mathrm{ ) where:
C
c}\begin{array}{lllllll}{1.0}&{3.0}&{0.0}&{0.0}&{1.0}&{7.0}
c 2.0
c A = 0.0 0.0 6.0 0.0 x = 3.0 b = 18.0
c
C
    data colstr / 1, 3, 6, 7, 9 /
        data rowind / 1, 2, 1, 2, 4, 3, 2, 4 /
        data values / 1.0d0, 2.0d0, 3.0d0, 4.0d0, 5.0d0, 6.0d0, 7.0d0,
        & 8.0d0 /
        data rhs / 7.0d0, 38.0d0, 18.0d0, 42.0d0 /
        data xexpct / 1.0d0, 2.0d0, 3.0d0, 4.0d0 /
C
c initialize solver
C
        mtxtyp= 'su'
        pivot = 'n'
        neqns = 4
        outunt = 6
        msglvl = 0
C
c call regular interface
C
        call dgssin ( mtxtyp, pivot, neqns , colstr, rowind,
        & outunt, msglvl, handle, ier )
        if ( ier .ne. 0 ) goto 110
C
c ordering and symbolic factorization
c
        ordmthd = 'mmd'
        call dgssor ( ordmthd, handle, ier )
        if ( ier .ne. 0 ) goto 110
C
c numeric factorization
C
    call dgssfa ( neqns, colstr, rowind, values, handle, ier )
```

```
    if ( ier .ne. 0 ) goto 110
C
c solution
C
    nrhs = 1
    ldrhs = 4
    call dgsssl ( nrhs, rhs, ldrhs, handle, ier )
    if ( ier .ne. 0 ) goto 110
C
c deallocate sparse solver storage
C
    call dgssda ( handle, ier )
    if ( ier .ne. 0 ) goto 110
C
c print values of sol
c
    write(6,200) 'i', 'rhs(i)', 'expected rhs(i)', 'error'
    do i = 1, neqns
            write(6,300) i, rhs(i), xexpct(i), (rhs(i)-xexpct(i))
    enddo
    stop
    1 1 0 \text { continue}
C
c call to sparse solver returns an error
c
    write ( 6 , 400 )
    & ' example: FAILED sparse solver error number = ', ier
    stop
    200 format(a5,3a20)
300 format(i5,3d20.12) ! i/sol/xexpct values
400 format(a60,i20) ! fail message, sparse solver error number
    end
my_system% f95 -dalign example_su.f -library=sunperf
my_system% a.out
    i rhs(i) expected rhs(i) error
    1 0.100000000000D+01 0.100000000000D+01 0.0000000000000D+00
    2 0.2000000000000D+01 0.2000000000000D+01 0.000000000000D+00
    3 0.3000000000000D+01 0.300000000000D+01 0.0000000000000D+00
    4 0.400000000000D+01 0.400000000000D+01 0.0000000000000D+00
```


## EXAMPLE 4 Solving an Unsymmetric System - Regular Interface

```
my_system% cat example_uu.f
        program example_uu
c
c This program is an example driver that calls the sparse solver.
c It factors and solves an unsymmetric system.
c
        implicit none
        integer neqns, ier, msglvl, outunt, ldrhs, nrhs
        character mtxtyp*2, pivot*1, ordmthd*3
        double precision handle(150)
        integer colstr(6), rowind(10)
        double precision values(10), rhs(5), xexpct(5)
        integer i
c
c Sparse matrix structure and value arrays. Unsummetric matrix A.
c Ax = b, (solve for }x\mathrm{ ) where:
c
c 1.llllllll
c 
```



```
c
c 
c
    data colstr / 1, 6, 7, 8, 9, 11 /
    data rowind / 1, 2, 3, 4, 5, 2, 3, 4, 2, 5 /
    data values / 1.0d0, 2.0d0, 3.0d0, 4.0d0, 5.0d0, 6.0d0, 7.0d0,
    & 8.0d0, 9.0d0, 10.0d0 /
        data rhs / 1.0d0, 59.0d0, 24.0d0, 36.0d0, 55.0d0 /
        data xexpct / 1.0d0, 2.0d0, 3.0d0, 4.0d0, 5.0d0 /
C
c initialize solver
c
        mtxtyp= 'uu'
        pivot = 'n'
        neqns = 5
        outunt = 6
        msglvl = 3
        call dgssin ( mtxtyp, pivot, neqns , colstr, rowind,
        & outunt, msglvl, handle, ier )
        if ( ier .ne. 0 ) goto 110
c
c ordering and symbolic factorization
c
    ordmthd = 'mmd'
```

```
            call dgssor ( ordmthd, handle, ier )
            if ( ier .ne. 0 ) goto 110
C
c numeric factorization
c
    call dgssfa ( neqns, colstr, rowind, values, handle, ier )
    if ( ier .ne. 0 ) goto 110
c
c solution
C
    nrhs = 1
    ldrhs = 5
    call dgsssl ( nrhs, rhs, ldrhs, handle, ier )
    if ( ier .ne. 0 ) goto 110
C
C deallocate sparse solver storage
c
    call dgssda ( handle, ier )
    if ( ier .ne. 0 ) goto 110
c
c print values of sol
c
    write(6,200) 'i', 'rhs(i)', 'expected rhs(i)', 'error'
    do i = 1, neqns
        write(6,300) i, rhs(i), xexpct(i), (rhs(i)-xexpct(i))
    enddo
    stop
    1 1 0 \text { continue}
C
c call to sparse solver returns an error
c
    write ( 6 , 400 )
    & ' example: FAILED sparse solver error number = ', ier
    stop
200 format(a5,3a20)
300 format(i5,3d20.12) ! i/sol/xexpct values
400 format(a60,i20) ! fail message, sparse solver error number
    end
my_system% f95 -dalign example_uu.f -library=sunperf
my_system% a.out
i rhs(i) expected rhs(i) error
    1 0.100000000000D+01 0.100000000000D+01 0.000000000000D+00
    2 0.200000000000D+01 0.200000000000D+01 0.000000000000D+00
    3 0.300000000000D+01 0.300000000000D+01 0.0000000000000D+00
```

```
4 0.400000000000D+01 0.400000000000D+01 0.000000000000D+00
5 0.500000000000D+01 0.500000000000D+01 0.000000000000D+00
```

```
EXAMPLE 5 Calling SPSOLVE Routines from C
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <sys/time.h>
#include <sunperf.h>
int main() {
/*
    Sparse matrix structure and value arrays. Coefficient matrix
    is a general unsymmetric sparse matrix.
    Ax = b, (solve for x) where:
\(A=\)\begin{tabular}{rrrrrrr}
1.0 & 0.0 & 7.0 & 9.0 & 0.0 & 1.0 & 17.0 \\
2.0 & 4.0 & 0.0 & 0.0 & 0.0 & 1.0 & 6.0 \\
0.0 & 5.0 & 8.0 & 0.0 & 0.0 & \(x=1.0\) & \(b=13.0\) \\
0.0 & 0.0 & 0.0 & 10.0 & 11.0 & 1.0 & 21.0 \\
3.0 & 6.0 & 0.0 & 0.0 & 12.0 & 1.0 & 21.0
\end{tabular}
*/
/* Array indices must be one-based for calling SPSOLVE routines */
int colstr[] = {1, 4, 7, 9, 11, 13};
int rowind[] ={1, 2, 5, 2, 3, 5, 1, 3, 1, 4, 4, 5};
double values[] = {1.0, 2.0, 3.0, 4.0, 5.0, 6.0,
                    7.0, 8.0, 9.0, 10.0, 11.0, 12.0};
double rhs[] = {17.0, 6.0, 13.0, 21.0, 21.0};
double xexpct[] = {1.0, 1.0, 1.0, 1.0, 1.0};
    int n = 5, nnz = 12, nrhs = 1, msglvl = 0, outunt = 6, ierr,
            i,j,k, int_ierr;
    double t[4], handle[150];
    char type[] = "uu", piv = 'n';
/* Last two parameters in argument list indicate lengths of
    * character arguments type and piv
    */
    dgssin_(type, &piv, &n, colstr, rowind, &outunt, &msglvl,
                handle, &ierr,2,1);
    if (ierr != 0) {
        int_ierr = ierr;
        printf("dgssin err = %d\n", int_ierr);
        return -1;
```

```
    }
    char ordmth[] = "mmd";
    dgssor_(ordmth, handle, &ierr, 3);
    if (ierr != 0) {
        int_ierr = ierr;
        printf("dgssor err = %d\n", int_ierr);
        return -1;
        }
        dgssfa_(&n, colstr, rowind, values, handle, &ierr);
        if (ierr != 0) {
        int_ierr = ierr;
        printf("dgssfa err = %d\n", int_ierr);
        return -1;
        }
        dgsssl_(&nrhs, rhs, &n, handle, &ierr);
        if (ierr != 0) {
        int_ierr = ierr;
        printf("dgssssl err = %d\n", int_ierr);
        return -1;
        }
        printf("i computed solution expected solution\n");
        for (i=0; i<n; i++)
        printf("%d %lf %lf\n", i,rhs[i], 1.0);
}
my_system% cc -m32 -xmemalign=8s dr.c -library=sunperf
my_system% ./a.out
    computed solution expected solution
            1.000000 1.000000
            1.000000 1.000000
            1.000000 1.000000
            1.000000 1.000000
            1.000000 1.000000
```


## SuperLU Interface

SuperLU has two driver routines, simple and expert, that can be called to completely solve a general unsymmetric sparse system in a similar manner to the one-call interface in SPSOLVE. These and other SuperLU user-callable routines are available in single precision, double precision, complex and double complex data types. Single precision names of all external routines are listed in the following tables. Man pages (section 3P) are available for these
routines. Also see the man page of SuperMatrix(3P) for a description of the sparse matrix data structure that is used in the application.

TABLE 5 SuperLU Computational Routines

| Routine | Description |
| :--- | :--- |
| sgstrf | Computes factorization |
| sgssvx | Factorizes and solves (expert driver) |
| sgssv | Factorizes and solves (simple driver) |
| sgstrs | Computes triangular solve |
| sgsrfs | Improves computed solution; provides error bounds |
| slangs | Computes one-norm, Frobenius-norm, or infinity-norm |
| sgsequ | Computes row and column scalings |
| sgscon | Estimates reciprocal of condition number |
| slaqgs | Equilibrates a general sparse matrix |

TABLE 6 SuperLU Utility Routines

| Routine | Description |
| :--- | :--- |
| LUSolveTime | Returns time spent in solve stage |
| LUFactTime | Returns time spent in factorization stage |
| LUFactFlops | Returns number of floating point operations in factorization stage |
| LUSolveFlops | Returns number of floating point operations in solve stage |
| sQuerySpace | Returns information on the memory statistics |
| sp_ienv | Returns specified machine dependent parameter |
| sPrintPerf | Prints statistics collected by the computational routines |
| set_default_options | Sets parameters that control solver behavior to default options |
| StatInit | Allocates and initializes structure that stores performance statistics |
| StatFree | Frees structure that stores performance statistics |
| Destroy_Dense_Matrix | Deallocates a SuperMatrix in dense format |
| Destroy_SuperNode_Matrix | Deallocates a SuperMatrix in supernodal format |
| Destroy_CompCol_Matrix | Deallocates a SuperMatrix in compressed sparse column format |
| Destroy_CompCol_Permuted | Deallocates a SuperMatrix in permuted compressed sparse column |
| fermat |  |
| Destroy_SuperMatrix_Store | Deallocates actual storage that stores matrix in a SuperMatrix |
| sCopy_CompCol_Matrix | Copies a SuperMatrix in compressed sparse column format |
| sCreate_CompCol_Matrix | Allocates a SuperMatrix in compressed sparse column format |
| sCreate_Dense_Matrix | Allocates a SuperMatrix in dense format |
| sCreate_CompRow_Matrix | Allocates a SuperMatrix in compressed sparse row format |


| Routine | Description |
| :--- | :--- |
| sCreate_SuperNode_Matrix | Allocates a SuperMatrix in supernodal format |
| sp_preorder | Permutes columns of original sparse matrix |
| sp_sgemm | Multiplies a SuperMatrix by a dense matrix |

## Calling SuperLU from C

SuperLU routines are written in C. Therefore, column- and row-related indices must be zerobased. In the following example, double precision simple driver dgssv () is called to compute factors L and U and to solve for the solution matrix.

## EXAMPLE 6 SuperLU Simple Driver

```
#include <stdio.h>
#include <sunperf.h>
#define M 5
#define N 5
int main(int argc, char *argv[])
{
    SuperMatrix A, L, U, B1, B2;
    int perm_r[M]; /* row permutations from partial pivoting */
    int perm_c[N]; /* column permutation vector */
    int info, i;
    superlu_options_t options;
    SuperLUStat_t stat;
    trans_t trans = NOTRANS;
    printf("Example code calling SuperLU simple driver to factor a \n");
    printf("general unsymmetric matrix and solve two right-hand-side matrices\n");
    /* the matrix in Harwell-Boeing format. */
    int m = M;
    int n = M;
    int nnz = 12;
    double *dp;
    /* nonzeros of A, column-wise */
    double a[] = {1.0, 2.0, 3.0, 4.0, 5.0, 6.0,
                    7.0, 8.0, 9.0, 10.0, 11.0, 12.0};
    /* row index of nonzeros */
    int asub[] = {0, 1, 4, 1, 2, 4, 0, 2, 0, 3, 3, 4};
    /* column pointers */
    int xa[] = {0, 3, 6, 8, 10, 12};
```

```
/* Create Matrix A in the format expected by SuperLU */
dCreate_CompCol_Matrix(&A, m, n, nnz, a, asub, xa, SLU_NC, SLU_D, SLU_GE);
int nrhs = 1;
double rhs1[] = {17.0, 6.0, 13.0, 21.0, 21.0};
double rhs2[] = {17*.3, 6*.3, 13*.3, 21*.3, 21*.3};
/* right-hand side matrix B1, B2 */
dCreate_Dense_Matrix(&B1, m, nrhs, rhs1, m, SLU_DN, SLU_D, SLU_GE);
dCreate_Dense_Matrix(&B2, m, nrhs, rhs2, m, SLU_DN, SLU_D, SLU_GE);
/* set options that control behavior of solver to default parameters */
set_default_options(&options);
options.ColPerm = NATURAL
/* Initialize the statistics variables. */
StatInit(&stat);
/* factor input matrix and solve the first right-hand-side matrix */
dgssv(&options, &A, perm_c, perm_r, &L, &U, &B1, &stat, &info);
printf("\nsolution matrix B1:\n");
dp = (double *) (((NCformat *)B1.Store)->nzval);
printf(" i rhs[i] expected\n");
for (i=0; i<M; i++)
    printf("%5d %7.4lf %7.4lf\n", i, dp[i], 1.0);
printf("Factor time = %8.2e sec\n", stat.utime[FACT]);
printf("Solve time = %8.2e sec\n\n\n", stat.utime[SOLVE]);
/* solve the second right-hand-side matrix */
dgstrs(trans, &L, &U, perm_c, perm_r, &B2, &stat, &info);
printf("solution matrix B2:\n");
dp = (double *) (((NCformat *)B2.Store)->nzval);
printf(" i rhs[i] expected\n");
for (i=0; i<M; i++)
    printf("%5d %7.4lf %7.4lf\n", i, dp[i], 0.3);
printf("Solve time = %8.2e sec\n", stat.utime[SOLVE]);
StatFree(&stat);
Destroy_CompCol_Matrix(&A);
Destroy_SuperMatrix_Store(&B1);
Destroy_SuperMatrix_Store(&B2);
Destroy_SuperNode_Matrix(&L);
Destroy_CompCol_Matrix(&U);
```

\}

Running the above example:

```
my_system% cc -xmemalign=8s simple.c -library=sunperf
my_system% a.out
Example code calling SuperLU simple driver to factor a
general unsymmetric matrix and solve two right-hand-side matrices
solution matrix B1:
        i rhs[i] expected
        0 1.0000 1.0000
        1 1.0000 1.0000
        2 1.0000 1.0000
        3 1.0000 1.0000
        4 1.0000 1.0000
Factor time = 5.43e-02 sec
Solve time = 6.76e-03 sec
solution matrix B2:
\begin{tabular}{ccc}
\(i\) & rhs[i] & expected \\
0 & 0.3000 & 0.3000 \\
1 & 0.3000 & 0.3000 \\
2 & 0.3000 & 0.3000 \\
3 & 0.3000 & 0.3000 \\
4 & 0.3000 & 0.3000
\end{tabular}
```


## EXAMPLE 7 SuperLU Expert Driver

```
#include <stdio.h>
#include <sunperf.h>
#define M 5
#define N 5
#define NRHS 1
int main(int argc, char *argv[])
{
    SuperMatrix A, L, U, B, X;
    int perm_r[M]; /* row permutations from partial pivoting */
    int perm_c[N]; /* column permutation vector */
    int etree[N]; /* elimination tree */
    double ferr[NRHS]; /* estimated forward error bound */
    double berr[NRHS]; /* component-wise relative backward error */
    double C[N], R[M]; /* column and row scale factors */
    double rpg, rcond;
    char equed[1]; /* Specifies the form of equilibration that was done */
    double *work, *dp; /* user-supplied workspace */
```

```
int lwork = 0; /* 0 for workspace to be allocated by system malloc */
int info, i;
superlu_options_t options;
SuperLUStat_t stat;
mem_usage_t mem_usage;
printf("Example code calling SuperLU expert driver\n\n");
/* the matrix in Harwell-Boeing format. */
int m = M;
int n = M;
int nnz = 12;
/* nonzeros of A, column-wise */
double a[] = {1.0, 2.0, 3.0, 4.0, 5.0, 6.0,
    7.0, 8.0, 9.0, 10.0, 11.0, 12.0};
/* row index of nonzeros */
int asub[] = {0, 1, 4, 1, 2, 4, 0, 2, 0, 3, 3, 4};
/* column pointers */
int xa[] = {0, 3, 6, 8, 10, 12};
int nrhs = NRHS;
double rhs[] = {17.0, 6.0, 13.0, 21.0, 21.0};
/* Create Matrix A in the format expected by SuperLU */
dCreate_CompCol_Matrix(&A, m, n, nnz, a, asub, xa, SLU_NC, SLU_D, SLU_GE);
/* right-hand-side matrix B */
dCreate_Dense_Matrix(&B, m, nrhs, rhs, m, SLU_DN, SLU_D, SLU_GE);
/* solution matrix X */
dCreate_Dense_Matrix(&X, m, nrhs, rhs, m, SLU_DN, SLU_D, SLU_GE);
set_default_options(&options);
options.ColPerm = NATURAL;
/* Initialize the statistics variables. */
StatInit(&stat);
dgssvx(&options, &A, perm_c, perm_r, etree, equed, R, C, &L, &U, work, lwork,
            &B, &X, &rpg, &rcond, ferr, berr, &mem_usage, &stat, &info);
dp = (double *) (((NCformat *)X.Store)->nzval);
printf(" i rhs[i] expected\n");
for (i=0; i<M; i++)
    printf("%5d %7.4lf %7.4lf\n",
            i, dp[i], 1.0);
printf("Factor time = %8.2e sec\n", stat.utime[FACT]);
printf("Solve time = %8.2e sec\n", stat.utime[SOLVE]);
StatFree(&stat);
Destroy_CompCol_Matrix(&A);
```

Destroy_SuperMatrix_Store(\&B);
Destroy_SuperNode_Matrix(\&L);
Destroy_CompCol_Matrix(\&U); \}

Running the above example:

```
my_system% cc -xmemalign=8s expert.c -library=sunperf
my_system% a.out
Example code calling SuperLU expert driver
\begin{tabular}{ccr} 
i & rhs[i] & expected \\
0 & 1.0000 & 1.0000 \\
1 & 1.0000 & 1.0000 \\
2 & 1.0000 & 1.0000 \\
3 & 1.0000 & 1.0000 \\
4 & 1.0000 & 1.0000 \\
tor time \(=1.25 \mathrm{e}-03 \mathrm{sec}\) \\
ve time & \(=1.70 \mathrm{e}-04 \mathrm{sec}\)
\end{tabular}
```


## Calling SuperLU from Fortran

The simplest way to call SuperLU from Fortran is through the SPSOLVE interface. SuperLU can be selected to solve an unsymmetric coefficient matrix through input argument MTXTYP of routine DGSSIN ( ), which is the initialization routine in SPSOLVE. The same argument also exists in the one-call interface routine DGSSFS ().

Valid options for MTXTYP are listed in the following table. To invoke SuperLU, select 's0' or 'S0' as matrix type. Since SPSOLVE is Fortran-based, all column and row indices associated with the input matrix should be one-based. However, if SuperLU is invoked through DGSSIN ( ) or DGSSFS ( ) (by setting MTXTYP = 's0' or 'S0'), these indices must be zero-based.

TABLE 7 Matrix Type Options for DGSSIN() and DGSSFS()

| Option | Type of Matrix | Solver |
| :--- | :--- | :--- |
| 'sp' or 'SP' | symmetric structure, positive-definite values | SPSOLVE |
| 'ss' or 'SS' | symmetric structure, symmetric values | SPSOLVE |
| 'su' or 'SU' | symmetric structure, unsymmetric values | SPSOLVE |
| 'uu' or 'UU' | unsymmetric structure, unsymmetric values | SPSOLVE |
| 's0' or 'S0' | unsymmetric structure, unsymmetric values | SuperLU |

A call to routine $\operatorname{DGSSOR}()$ must follow $\operatorname{DGSSIN}()$ to perform fill-reduce ordering and symbolic factorization. A character argument (ORDMTHD) is used to select the desired ordering method.

This argument also exists in the one-call interface routine DGSSFS(). Valid ordering methods for SPSOLVE and SuperLU are listed in the following table. You can also provide a particular ordering to the solver by calling DGSSUO ( ) in place of $\operatorname{DGSSOR}$ ( ). The input permutation array must be zero-based.

TABLE 8 Matrix Ordering Options for DGSSOR() and DGSSFS()

| Option | Ordering Method | Solver |
| :--- | :--- | :--- |
| 'nat' or 'NAT' | natural ordering (no ordering) | SPSOLVE, SuperLU |
| 'mmd' or 'MMD' | minimum degree on A'*A (default) | SPSOLVE, SuperLU |
| 'gnd' or 'GND' | general nested dissection | SPSOLVE |
| 'spm' or 'SPM' | Minimum degree ordering on A'+A | SuperLU |
| 'sam' or 'SAM' | Approximate minimum degree column | SuperLU |

As shown above, the general nested dissection method is not available in SuperLU. On the other hand, the minimum degree ordering on $\mathrm{A}^{\prime}+\mathrm{A}$ and approximate minimum degree column ordering are not available in SPSOLVE.

## SuperLU Examples

The following code examples show how SuperLU can be selected through the regular interface and the one-call interface of SPSOLVE to factorize and solve a general unsymmetric system of equations.

EXAMPLE 8 Invoking SuperLU Through SPSOLVE Regular Interface
program SLU
c This program is an example driver that calls the regular interface of SPSOLVE
c to invoke SuperLU to factor and solve a general unsymmetric system.
implicit none
integer neqns, ier, msglvl, outunt, ldrhs, nrhs, i
character mtxtyp*2, pivot*1, ordmthd*3
double precision handle(150)
integer colstr(6), rowind(12)
double precision values(12), rhs(5), xexpct(5)
c Sparse matrix structure and value arrays. Coefficient matrix
c is a general unsymmetric sparse matrix.
c $\quad A x=b$, (solve for $x$ ) where:

| c |  | 1.0 | 0.0 | 7.0 | 9.0 | 0.0 | 1.0 | 17.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C |  | 2.0 | 4.0 | 0.0 | 0.0 | 0.0 | 1.0 | 6.0 |
| C | $A=$ | 0.0 | 5.0 | 8.0 | 0.0 | 0.0 | $x=1.0$ | $\mathrm{b}=13.0$ |
| C |  | 0.0 | 0.0 | 0.0 | 10.0 | 11.0 | 1.0 | 21.0 |
| C |  | 3.0 | 6.0 | 0.0 | 0.0 | 12.0 | 1.0 | 21.0 |

C Array indices must be zero-based for calling SuperLU data colstr / 0, 3, 6, 8, 10, 12 / data rowind / 0, 1, 4, 1, 2, 4, 0, 2, 0, 3, 3, 4/ data values / 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, \$
$7.0,8.0,9.0,10.0,11.0,12.0 /$
data rhs / 17.0, 6.0, 13.0, 21.0, 21.0 / data xexpct / 1.0d0, 1.0d0, 1.0d0, 1.0d0, 1.0d0 /
c initialize solver
mtxtyp= 's0'
pivot = 'n'
neqns $=5$
outunt $=6$
msglvl $=0$
c call regular interface
call dgssin(mtxtyp, pivot, neqns , colstr, rowind,outunt, msglvl,
\& handle, ier)
if ( ier .ne. 0 ) goto 110
c ordering and symbolic factorization
ordmthd = 'mmd'
call dgssor(ordmthd, handle, ier)
if ( ier .ne. 0 ) goto 110
c numeric factorization
call dgssfa ( neqns, colstr, rowind, values, handle, ier ) if ( ier .ne. 0 ) goto 110
c solution
nrhs $=1$
ldrhs $=5$
call dgsssl ( nrhs, rhs, ldrhs, handle, ier )
if ( ier .ne. 0 ) goto 110
c deallocate sparse solver storage
call dgssda ( handle, ier )
if ( ier .ne. 0 ) goto 110
c print values of sol
write(6,200) 'i', 'rhs(i)', 'expected rhs(i)', 'error'
do $i=1$, neqns

```
            write(6,300) i, rhs(i), xexpct(i), (rhs(i)-xexpct(i))
        enddo
        stop
    110 continue
c call to sparse solver returns an error
    write ( 6 , 400 )
        & ' example: FAILED sparse solver error number = ', ier
            stop
200 format(4x,a1,3x,a6,3x,a15,4x,a6)
300 format(i5,3x,f5.2,7x,f5.2,8x,e10.2) ! i/sol/xexpct values
400 format(a60,i20) ! fail message, sparse solver error number
    end
```

Running the above example:
my_system\% f95 -dalign slu.f -library=sunperf my_system\% a.out

| rhs(i) | expected rhs(i) | error |
| :---: | :---: | :---: |
| 1.00 | 1.00 | $0.00 \mathrm{E}+00$ |
| 1.00 | 1.00 | -0.33E-15 |
| 1.00 | 1.00 | $0.22 \mathrm{E}-15$ |
| 1.00 | 1.00 | -0.11E-15 |
| 1.00 | 1.00 | 0.22E-15 |

EXAMPLE 9 Invoking SuperLU through One-Call SPSOLVE Interface

```
program SLU_SINGLE
```

c This program is an example driver that calls the regular interface of SPSOLVE
c to invoke SuperLU to factor and solve a general unsymmetric system.

| implicit none <br> integer <br> character <br> double precision | neqns, ier, msglvl, outunt, ldrhs, nrhs, i |
| :--- | :--- |
| integer | colstre(150) |
| double precision | values(12), rowind(12) |

c Sparse matrix structure and value arrays. Coefficient matrix
c is a general unsymmetric sparse matrix.
c $\quad A x=b$, (solve for $x$ ) where:

| C | 1.0 | 0.0 | 7.0 | 9.0 | 0.0 | 1.0 | 17.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | 2.0 | 4.0 | 0.0 | 0.0 | 0.0 | 1.0 | 6.0 |
| c | 0.0 | 5.0 | 8.0 | 0.0 | 0.0 | 1.0 | 13.0 |

```
c 0.0}00.00.0.0 10.0 11.0 1.0 1.0 % 21.0 
c }\begin{array}{llllllll}{3.0}&{6.0}&{0.0}&{0.0}&{12.0}&{1.0}&{21.0}
c Array indices must be zero-based for calling SuperLU
        data colstr / 0, 3, 6, 8, 10, 12 /
        data rowind / 0, 1, 4, 1, 2, 4, 0, 2, 0, 3, 3, 4 /
        data values / 1.0, 2.0, 3.0, 4.0, 5.0, 6.0,
        $ 7.0, 8.0, 9.0, 10.0, 11.0, 12.0 /
        data rhs / 17.0, 6.0, 13.0, 21.0, 21.0 /
        data xexpct / 1.0d0, 1.0d0, 1.0d0, 1.0d0, 1.0d0 /
c initialize solver
    mtxtyp= 's0'
    pivot = 'n'
    neqns = 5
    outunt = 6
    msglvl = 0
    ordmthd = 'mmd'
    nrhs = 1
    ldrhs = 5
c One-call routine of SPSOLVE
        call dgssfs (mtxtyp, pivot, neqns , colstr, rowind,
        & values, nrhs , rhs, ldrhs , ordmthd,
        & outunt, msglvl, handle, ier)
        if ( ier .ne. 0 ) goto 110
c deallocate sparse solver storage
        call dgssda ( handle, ier )
        if ( ier .ne. 0 ) goto 110
c print values of sol
        write(6,200) 'i', 'rhs(i)', 'expected rhs(i)', 'error'
        do i = 1, neqns
            write(6,300) i, rhs(i), xexpct(i), (rhs(i)-xexpct(i))
        enddo
        stop
    1 1 0 \text { continue}
c call to sparse solver returns an error
            write ( 6 , 400 )
        & ' example: FAILED sparse solver error number = ', ier
            stop
        200 format(4x,a1,3x,a6,3x,a15,4x,a6)
        300 format(i5,3x,f5.2,7x,f5.2,8x,e10.2) ! i/sol/xexpct values
        400 format(a60,i20) ! fail message, sparse solver error number
            end
```

Running the above example:

```
my_system% f95 -dalign slu_single.f -library=sunperf
my_system% a.out
\begin{tabular}{lcrr} 
i & rhs(i) & expected & rhs(i)
\end{tabular}\(\quad\) error
```


## References for Sparse BLAS and Solver

The following books and papers provide additional information for the sparse BLAS and sparse solver routines.

1. D.S. Dodson, R.G. Grimes, and J.G. Lewis, Sparse Extensions to the Fortran Basic Linear Algebra Subprograms, ACM Transactions on Mathematical Software, June 1991, Vol 17, No. 2.
2. A. George and J. W-H. Liu, Computer Solution of Large Sparse Positive Definite Systems, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1981.
3. E. Ng and B. W. Peyton, Block Sparse Cholesky Algorithms on Advanced Uniprocessor Computers, SIAM M. Sci Comput., 14:1034-1056, 1993.
4. Ian S. Duff, Roger G. Grimes and John G. Lewis, User's Guide for the Harwell-Boeing Sparse Matrix Collection (Release I), Technical Report TR/PA/92/86, CERFACS, Lyon, France, October 1992.
5. J. W. Demmel, J. R. Gilbert, and X. S. Li, SuperLU User's Guide, Technical report LBNL44289.
6. X. S. Li, An Overview of SuperLU: Algorithms, Implementation, and User Interface, ACM Transactions on Mathematical Software, 2004.
7. 7. J. W. Demmel, S. C. Eisenstat, J. R. Gilbert, X. S. Li, J. W. H. Liu, A supernodal approach to sparse partial pivoting, SIAM J. Matrix Analysis and Applications, Vol 20, No. 3, 1999, pp. 720-755.

## Using Oracle Developer Studio Performance Library Signal Processing Routines

The discrete Fourier transform (DFT) has always been an important analytical tool in many areas of science and engineering. However, it was not until the development of the fast Fourier transform (FFT) that the DFT became widely used. This is because the DFT requires $\mathrm{O}\left(N^{2}\right)$ computations, while the FFT only requires $\mathrm{O}\left(N \log _{2} N\right)$ operations.

Oracle Developer Studio Performance Library contains a set of routines that computes the FFT, related FFT operations, such as convolution and correlation, and trigonometric transforms.

This chapter is divided into the following three sections.

- Forward and Inverse FFT Routines
- Sine and Cosine Transforms
- Convolution and Correlation

Each section includes examples that show how the routines might be used.

Tip - For information on the Fortran 95 and C interfaces and types of arguments used in each routine, see the section 3P man pages for the individual routines. Routine names for man pages must be lowercase.

For example, to display the man page for the SFFTC routine, use the following command specifying the routine name in lowercase:
\% man -s 3P sfftc
For an overview of the FFT routines:
\% man -s 3P fft

## Forward and Inverse FFT Routines

The following tables list the names of the FFT routines and their calling sequence. Double precision routine names are in square brackets. See the individual man pages for detailed information on the data type and size of the arguments.

- Table 9, "FFT Linear Routines and Their Arguments," on page 78
- Table 10, "FFT Two-Dimensional Routines and Their Arguments," on page 78
- Table 11, "FFT Three-Dimensional Routines and Their Arguments," on page 78

TABLE $9 \quad$ FFT Linear Routines and Their Arguments

| Routine Name | Arguments |
| :---: | :---: |
| CFFTS [ZFFTD] | (OPT, N1, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, ERR) |
| SFFTC [DFFTZ] | (OPT, N1, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, ERR) |
| CFFTSM [ ZFFTDM] | (OPT, N1, N2, SCALE, X, LDX1, Y, LDY1, TRIGS, IFAC, WORK, LWORK, ERR) |
| SFFTCM [DFFTZM] | (OPT, N1, N2, SCALE, X, LDX1, Y, LDY1, TRIGS, IFAC, WORK, LWORK, ERR) |
| CFFTC [ZFFTZ] | (OPT, N1, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, ERR) |
| CFFTCM [ZFFTZM] | (OPT, N1, N2, SCALE, X, LDX1, Y, LDY1, TRIGS, IFAC, WORK, LWORK, ERR) |

TABLE 10 FFT Two-Dimensional Routines and Their Arguments

| Routine Name | Arguments |
| :--- | :--- |
| CFFTS2 [ZFFTD2] | (OPT, N1, N2, SCALE, X, LDX1, Y, LDY1, TRIGS, IFAC, <br>  <br> WORK, LWORK, ERR) |
| CFFTC2 [DFFTZ2] | (OPT, N1, N2, SCALE, X, LDX1, Y, LDY1, TRIGS, IFAC, <br> WORK, LWORK, ERR) |
|  | (OPT, N1, N2, SCALE, X, LDX1, Y, LDY1, TRIGS, IFAC, <br> [ZFFTZ2] |

TABLE 11 FFT Three-Dimensional Routines and Their Arguments

| Routine Name | Arguments |
| :--- | :--- |
| CFFTS3 [ZFFTD3] | (OPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, |
|  | LDY2, TRIGS, IFAC, WORK, LWORK, ERR) |


| Routine Name | Arguments |
| :--- | :--- |
| SFFTC3 [DFFTZ3] | (OPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, |
|  | LDY2, TRIGS, IFAC, WORK, LWORK, ERR) |
| CFFTC3 [ZFFTZ3] | (OPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, |
|  | LDY2, TRIGS, IFAC, WORK, LWORK, ERR) |

Oracle Developer Studio Performance Library FFT routines use the following arguments.

- OPT: Flag indicating whether the routine is called to initialize or to compute the transform.
- N1, N2, N3: Problem dimensions for one, two, and three dimensional transforms.
- X : Input array where X is of type COMPLEX if the routine is a complex-to-complex transform or a complex-to-real transform. X is of type REAL for a real-to-complex transform.
- $Y$ : Output array where $Y$ is of type COMPLEX if the routine is a complex-to-complex transform or a real-to-complex transform. $Y$ is of type REAL for a complex-to-real transform.
- LDX1, LDX2 and LDY1, LDY2: LDX1 and LDX2 are the leading dimensions of the input array, and LDY1 and LDY2 are the leading dimensions of the output array. The FFT routines allow the output to overwrite the input, which is an in-place transform, or to be stored in a separate array apart from the input array, which is an out - of - place transform. In complex-tocomplex transforms, the input data is of the same size as the output data. However, real-to-complex and complex-to-real transforms have different memory requirements for input and output data. Care must be taken to ensure that the input array is large enough to accommodate the transform results when computing an in-place transform.
- TRIGS: Array containing the trigonometric weights.
- IFAC: Array containing factors of the problem dimensions. The problem sizes are as follows:
- Linear FFT: Problem size of dimension N1
- Two-dimensional FFT: Problem size of dimensions N1 and N2
- Three-dimensional FFT: Problem size of dimensions N1, N2, and N3

While N1, N2, and N3 can be of any size, a real-to-complex or a complex-to-real transform can be computed most efficiently when

$$
N 1, N 2, N 3=2^{p} \times 3^{q} \times 4^{r} \times 5^{s}
$$

and a complex-to-complex transform can be computed most efficiently when

$$
N 1, N 2, N 3=2^{p} \times 3^{q} \times 4^{r} \times 5^{s} \times 7^{t} \times 11^{u} \times 13^{v}
$$

where $p, q, r, s, t, u$, and $v$ are integers and $p, q, r, s, t, u, v \geq 0$.

- WORK: Workspace whose size depends on the routine and the number of threads that are being used to compute the transform if the routine is parallelized.
- LWORK: Size of workspace. If LWORK is zero, the routine will allocate a workspace with the required size.
- SCALE: A scalar with which the output is scaled. Occasionally in literature, the inverse transform is defined with a scaling factor of $1 / \mathrm{N} 1$ for one-dimensional transforms, 1/ $(\mathrm{N} 1 \times \mathrm{N} 2)$ for two-dimensional transforms, and $1 /(\mathrm{N} 1 \times \mathrm{N} 2 \times \mathrm{N} 3)$ for three-dimensional transforms. In such case, the inverse transform is said to be normalized. If a normalized FFT is followed by its inverse FFT, the result is the original input data. The Oracle Developer Studio Performance Library FFT routines are not normalized. However, normalization can be done easily by calling the inverse FFT routine with the appropriate scaling factor stored in SCALE.
- ERR: A flag returning a nonzero value if an error is encountered in the routine and zero otherwise.


## Linear FFT Routines

Linear FFT routines compute the FFT of real or complex data in one dimension only. The data can be one or more complex or real sequences. For a single sequence, the data is stored in a vector. If more than one sequence is being transformed, the sequences are stored column-wise in a two-dimensional array and a one-dimensional FFT is computed for each sequence along the column direction. The linear forward FFT routines compute:

$$
X(k)=\sum_{n=0}^{N l-1} x(n) e^{\frac{-2 \pi i n k}{N l}}, k=0, \ldots, N l-1, \quad \text { where } i=\sqrt{-1}
$$

Or expressed in polar form:

$$
X(k)=\sum_{n=0}^{N l-1} x(n)\left(\cos \left(\frac{2 \pi n k}{N l}\right)-i \sin \left(\frac{2 \pi n k}{N l}\right)\right), k=0, \ldots, N l-1
$$

The inverse FFT routines compute $x(n)=\sum_{k=0}^{N 1-1} X(k) e^{\frac{2 \pi i n k}{N}}, n=0, \ldots, N 1-1$,
In polar form:

$$
x(n)=\sum_{n=0}^{N-1} X(k)\left(\cos \left(\frac{2 \pi n k}{N 1}\right)+i \sin \left(\frac{2 \pi n k}{N 1}\right)\right), k=0, \ldots, N 1-1
$$

With the forward transform, if the input is one or more complex sequences of size $N 1$, the result will be one or more complex sequences, each consisting of $N 1$ unrelated data points. However, if the input is one or more real sequences, each containing $N 1$ real data points, the result will be one or more complex sequences that are conjugate symmetric. That is:

$$
X(k)=X *(N l-k), k=\frac{N l}{2}+1, \ldots, N l-1
$$

The imaginary part of $X(0)$ is always zero. If $N 1$ is even, the imaginary part of ${ }^{X\left(\frac{N I}{2}\right)}$ is also zero. Both zeros are stored explicitly. Because the second half of each sequence can be derived from the first half, only ${ }^{\frac{N I}{2}+1}$ complex data points are computed and stored in the output array. Here and elsewhere in this chapter, integer division is rounded down.

With the inverse transform, if an N1-point complex-to-complex transform is being computed, then $N 1$ unrelated data points are expected in each input sequence and $N 1$ data points will be returned in the output array. However, if an N1-point complex-to-real transform is being computed, only the first ${ }^{\frac{N I}{2}+1}$ complex data points of each conjugate symmetric input sequence are expected in the input, and the routine will return $N 1$ real data points in each output sequence.

For each value of $N 1$, either the forward or the inverse routine must be called to compute the factors of $N 1$ and the trigonometric weights associated with those factors before computing the actual FFT. The factors and trigonometric weights can be reused in subsequent transforms as long as $N 1$ remains unchanged.

The following table Table 12, "Single Precision Linear FFT Routines," on page 81 lists the single precision linear FFT routines and their purposes. For routines that have twodimensional arrays as input and output, Table 12, "Single Precision Linear FFT Routines," on page 81 also lists the leading dimension requirements. The same information applies to the corresponding double precision routines except that their data types are double precision and double complex. See Table 12, "Single Precision Linear FFT Routines," on page 81 for the mapping. See the individual man pages for a complete description of the routines and their arguments.

TABLE 12 Single Precision Linear FFT Routines

| Name | Purpose | Size and Type of <br> Input | Size and Type of <br> Output | Leading Dimension Requirements |
| :--- | :--- | :--- | :--- | :--- |
| SFFTC | OPT $=0$ initialization |  |  |  |
|  | OPT $=-1$ real-to-complex <br> forward linear FFT of a single <br> vector | Real |  |  |


| Name | Purpose | Size and Type of Input | Size and Type of Output | Leading Dimension Requirements |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SFFTC | OPT $=0$ initialization |  |  |  |  |
|  | OPT = 1 complex-to-real inverse linear FFT of single vector | $\frac{N I}{2}+1$ | $N 1$ Real |  |  |
|  |  | Complex |  |  |  |
| CFFTC | OPT $=0$ initialization |  |  |  |  |
|  | OPT = -1 complex-to-complex forward linear FFT of a single vector | $N 1,$ <br> Complex | $N 1,$ <br> Complex |  |  |
|  | OPT = 1 complex-to-complex inverse linear FFT of a single vector | $N 1,$ <br> Complex | $N 1,$ <br> Complex |  |  |
| SFFTCM | OPT $=0$ initialization |  |  |  |  |
|  | OPT $=-1$ real-to-complex forward linear FFT of M vectors | $N 1 \times M$, Real | $\left(\frac{N I}{2}+1\right) \times M$ | LDX1 $=2 \times$ LDY1 | LDX1 $\geq$ N1 |
|  |  |  | Complex |  |  |
| CFFTSM | OPT $=0$ initialization |  |  |  |  |
|  | OPT = 1 complex-to-real inverse linear FFT of M vectors | $\left(\frac{N I}{2}+1\right) \times M$ | $N 1 \times M$, Real | $\operatorname{LDX} 1 \geq \frac{N I}{2}+1$ | $\operatorname{LDX1} \geq \frac{N I}{2}+1$ |
|  |  | Complex |  | LDY1 $=2 \times$ LDX1 | LDY1 $\geq$ N1 |
| CFFTCM | OPT $=0$ initialization |  |  |  |  |
|  | OPT = -1 complex-to-complex forward linear FFT of M | $N 1 \times M$, | $N 1 \times M$, | LDX1 $\geq$ N1 | LDX1 $\geq N 1$ |
|  | vectors | Complex | Complex | LDY1 $\geq$ N1 | LDY1 $\geq$ N1 |
|  | OPT = 1 complex-to-complex inverse linear FFT of M | $N 1 \times M$, | $N 1 \times M$, | LDX1 $\geq$ N1 | LDX1 $\geq$ N1 |
|  |  | Complex | Complex | LDY1 $\geq$ N1 | LDY1 $\geq$ N1 |

Note - Note the following about the table Table 12, "Single Precision Linear FFT Routines," on page 81:

- LDX1 is the leading dimension of the input array.
- LDY1 is the leading dimension of the output array.
- N 1 is the first dimension of the FFT problem.
- N2 is the second dimension of the FFT problem.
- When calling routines with OPT $=0$ to initialize the routine, the only error checking that is done is to determine if $N 1<0$

The following example shows how to compute the linear real-to-complex and complex-to-real FFT of a set of sequences.

EXAMPLE 10 Linear Real-to-Complex FFT and Complex-to-Real FFT

```
my_system% cat testscm.f
    PROGRAM TESTSCM
    IMPLICIT NONE
    INTEGER :: LW, IERR, I, J, K, LDX, LDC
    INTEGER,PARAMETER :: N1 = 3, N2 = 2, LDZ = N1,
    $ LDC = N1, LDX = 2*LDC
        INTEGER, DIMENSION(:) :: IFAC(128)
    REAL :: SCALE
    REAL, PARAMETER :: ONE = 1.0
    REAL, DIMENSION(:) :: SW(N1), TRIGS(2*N1)
    REAL, DIMENSION(0:LDX-1,0:N2-1) :: X, V, Y
    COMPLEX, DIMENSION(0:LDZ-1, 0:N2-1) :: Z
* workspace size
    LW = N1
    SCALE = ONE/N1
    WRITE(*,*)
    $ 'Linear complex-to-real and real-to-complex FFT of a sequence'
    WRITE(*,*)
    X = RESHAPE(SOURCE = (/.1, .2, .3,0.0,0.0,0.0,7.,8.,9.,
    $ 0.0, 0.0, 0.0/), SHAPE=(/6,2/))
    V = X
    WRITE(*,*) 'X = '
    DO I = 0,N1-1
        WRITE(*,'(2(F4.1,2x))') (X(I,J), J = 0, N2-1)
    END DO
    WRITE(*,*)
* intialize trig table and compute factors of N1
    CALL SFFTCM(0, N1, N2, ONE, X, LDX, Z, LDZ, TRIGS, IFAC,
    $ SW, LW, IERR)
```

```
    IF (IERR .NE. 0) THEN
        PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
        STOP
    END IF
* Compute out-of-place forward linear FFT.
* Let FFT routine allocate memory.
            CALL SFFTCM(-1, N1, N2, ONE, X, LDX, Z, LDZ, TRIGS, IFAC,
            $ SW, 0, IERR)
            IF (IERR .NE. 0) THEN
            PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
            STOP
            END IF
            WRITE(*,*) 'out-of-place forward FFT of X:'
            WRITE(*,*)'Z ='
            DO I = 0, N1/2
            WRITE(*,'(2(A1, F4.1,A1,F4.1,A1,2x))') ('(',REAL(Z(I,J)),
    $ ',',AIMAG(Z(I,J)),')', J = 0, N2-1)
            END DO
    WRITE(*,*)
* Compute in-place forward linear FFT.
* X must be large enough to store N1/2+1 complex values
    CALL SFFTCM(-1, N1, N2, ONE, X, LDX, X, LDC, TRIGS, IFAC,
    $ SW, LW, IERR)
    IF (IERR .NE. 0) THEN
            PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
            STOP
    END IF
    WRITE(*,*) 'in-place forward FFT of X:'
    CALL PRINT_REAL_AS_COMPLEX(N1/2+1, N2, 1, X, LDC, N2)
    WRITE(*,*)
* Compute out-of-place inverse linear FFT.
    CALL CFFTSM(1, N1, N2, SCALE, Z, LDZ, X, LDX, TRIGS, IFAC,
    $ SW, LW, IERR)
        IF (IERR .NE. 0) THEN
            PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
            STOP
        END IF
        WRITE(*,*) 'out-of-place inverse FFT of Z:'
        DO I = 0, N1-1
            WRITE(*,'(2(F4.1,2X))') (X(I,J), J = 0, N2-1)
        END DO
        WRITE(*,*)
* Compute in-place inverse linear FFT.
    CALL CFFTSM(1, N1, N2, SCALE, Z, LDZ, Z, LDZ*2, TRIGS,
    $ IFAC, SW, 0, IERR)
        IF (IERR .NE. 0) THEN
            PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
```

```
        STOP
    END IF
    WRITE(*,*) 'in-place inverse FFT of Z:'
    CALL PRINT_COMPLEX_AS_REAL(N1, N2, 1, Z, LDZ*2, N2)
    WRITE(*,*)
    END PROGRAM TESTSCM
    SUBROUTINE PRINT_COMPLEX_AS_REAL(N1, N2, N3, A, LD1, LD2)
    INTEGER N1, N2, N3, I, J, K
    REAL A(LD1, LD2, *)
    DO K = 1, N3
        DO I = 1, N1
            WRITE(*,'(5(F4.1,2X))') (A(I,J,K), J = 1, N2)
        END DO
        WRITE (*,*)
    END DO
    END
    SUBROUTINE PRINT_REAL_AS_COMPLEX(N1, N2, N3, A, LD1, LD2)
    INTEGER N1, N2, N3, I, J, K
    COMPLEX A(LD1, LD2, *)
    DO K = 1, N3
        DO I = 1, N1
            WRITE(*,'(5(A1, F4.1,A1,F4.1,A1,2X))') ('(',REAL(A(I, J,K)),
    $
        END DO
        WRITE(*,*)
    END DO
    END
my_system% f95 -dalign testscm.f -xlibrary=sunperf
my_system% a.out
Linear complex-to-real and real-to-complex FFT of a sequence
X =
0.17.0
0.28.0
0.3 9.0
out-of-place forward FFT of X:
Z =
(0.6, 0.0) (24.0, 0.0)
(-0.2, 0.1) (-1.5, 0.9)
in-place forward FFT of X:
(0.6, 0.0) (24.0, 0.0)
(-0.2, 0.1) (-1.5, 0.9)
out-of-place inverse FFT of Z:
0.1 7.0
0.2 8.0
0.3 9.0
in-place inverse FFT of Z:
0.17.0
0.28.0
```


### 0.39 .0

Example 7-1 Notes:
The forward FFT of X is actually:

$$
Z=\begin{array}{cc}
(0.6,0.0) & (24.0,0.0) \\
(-0.2,0.1) & (-1.5,0.9) \\
(-0.2,0.1) & (-1.5,0.9)
\end{array}
$$

Because of symmetry, $\mathrm{Z}(2)$ is the complex conjugate of $\mathrm{Z}(1)$, and therefore only the first two $\frac{N I}{2}+1=2$ complex values are stored. For the in-place forward transform, SFFTCM is called with real array X as the input and output. Because SFFTCM expects the output array to be of type COMPLEX, the leading dimension of X as an output array must be as if X were complex. Since the leading dimension of real array X is $\mathrm{LDX}=2 \times \mathrm{LDC}$, the leading dimension of X as a complex output array must be LDC. Similarly, in the in-place inverse transform, CFFTSM is called with complex array Z as the input and output. Because CFFTSM expects the output array to be of type REAL, the leading dimension of Z as an output array must be as if Z were real. Since the leading dimension of complex array Z is LDZ, the leading dimension of Z as a real output array must be LDZ $\times 2$.

The following example Example 11, "Linear Complex-to-Complex FFT," on page 86 shows how to compute the linear complex-to-complex FFT of a set of sequences.

EXAMPLE 11 Linear Complex-to-Complex FFT

```
my_system% cat testccm.f
    PROGRAM TESTCCM
    IMPLICIT NONE
    INTEGER :: LDX1, LDY1, LW, IERR, I, J, K, LDZ1, NCPUS,
    $ USING_THREADS, IFAC(128)
    INTEGER, PARAMETER :: N1 = 3, N2 = 4, LDX1 = N1, LDZ1 = N1,
    $ LDY1 = N1+2
    REAL, PARAMETER :: ONE = 1.0, SCALE = ONE/N1
    COMPLEX :: Z(0:LDZ1-1,0:N2-1), X(0:LDX1-1,0:N2-1),
    $ Y(0:LDY1-1,0:N2-1)
    REAL :: TRIGS(2*N1)
    REAL, DIMENSION(:), ALLOCATABLE :: SW
* get number of threads
    NCPUS = USING_THREADS()
* workspace size
    LW = 2 * N1 * NCPUS
    WRITE(*,*)'Linear complex-to-complex FFT of one or more sequences'
```

```
    WRITE(*,*)
    ALLOCATE(SW(LW))
    X = RESHAPE(SOURCE =(/(.1,.2),(.3,.4),(.5,.6),(.7,.8),(.9,1.0),
$ (1.1,1.2),(1.3,1.4),(1.5,1.6),(1.7,1.8),(1.9,2.0),(2.1,2.2),
$ (1.2,2.0)/), SHAPE=(/LDX1,N2/))
    Z = X
    WRITE(*,*) 'X = '
    DO I = 0, N1-1
        WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))') ('(',REAL(X(I,J)),
    $ ',',AIMAG(X(I,J)),')', J = 0, N2-1)
    END DO
    WRITE(*,*)* intialize trig table and compute factors of N1
    CALL CFFTCM(0, N1, N2, SCALE, X, LDX1, Y, LDY1, TRIGS, IFAC,
    $ SW, LW, IERR)
    IF (IERR .NE. 0) THEN
        PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
        STOP
    END IF
* Compute out-of-place forward linear FFT.
* Let FFT routine allocate memory,
    CALL CFFTCM(-1, N1, N2, ONE, X, LDX1, Y, LDY1, TRIGS, IFAC,
    $ SW, 0, IERR)
    IF (IERR .NE. 0) THEN
        PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
            STOP
        END IF
* Compute in-place forward linear FFT. LDZ1 must equal LDX1
    CALL CFFTCM(-1, N1, N2, ONE, Z, LDX1, Z, LDZ1, TRIGS,
    $
        WRITE(*,*) 'in-place forward FFT of X:
        DO I = 0, N1-1
            WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))') ('(',REAL(Z(I,J)),
        $ ',',AIMAG(Z(I,J)),')', J = 0, N2-1)
        END DO
        WRITE(*,*)
        WRITE(*,*) 'out-of-place forward FFT of X:'
        WRITE(*,*) 'Y ='
        DO I = 0, N1-1
            WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))') ('(',REAL(Y(I,J)),
    $ ',',AIMAG(Y(I,J)),')', J = 0, N2-1)
        END DO
        WRITE(*,*)
* Compute in-place inverse linear FFT.
    CALL CFFTCM(1, N1, N2, SCALE, Y, LDY1, Y, LDY1, TRIGS, IFAC,
    $ SW, LW, IERR)
    IF (IERR .NE. 0) THEN
            PRINT*,'ROUTINE RETURN WITH ERROR CODE = ', IERR
            STOP
```

```
    END IF
    WRITE(*,*) 'in-place inverse FFT of Y:'
    WRITE(*,*) 'Y ='
    DO I = 0, N1-1
        WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))') ('(',REAL(Y(I,J)),
    $ ',',AIMAG(Y(I,J)),')', J = 0, N2-1)
    END DO
    DEALLOCATE(SW)
    END PROGRAM TESTCCM
my_system% f95 -dalign testccm.f -library=sunperf
my_system% a.out
Linear complex-to-complex FFT of one or more sequences
X =
(0.1, 0.2) ( 0.7, 0.8) ( 1.3, 1.4) ( 1.9, 2.0)
(0.3, 0.4) (0.9, 1.0) ( 1.5, 1.6) ( 2.1, 2.2)
(0.5, 0.6) ( 1.1, 1.2) ( 1.7, 1.8) ( 1.2, 2.0)
in-place forward FFT of X:
( 0.9, 1.2) ( 2.7, 3.0) ( 4.5, 4.8) ( 5.2, 6.2)
(-0.5,-0.1) (-0.5,-0.1) ( -0.5,-0.1) ( 0.4, -0.9)
( -0.1, -0.5) ( -0.1, -0.5) ( -0.1, -0.5) ( 0.1, 0.7)
out-of-place forward FFT of X:
Y =
( 0.9, 1.2) ( 2.7, 3.0) ( 4.5, 4.8) ( 5.2, 6.2)
( -0.5, -0.1) ( -0.5, -0.1) ( -0.5, -0.1) ( 0.4, -0.9)
( -0.1, -0.5) ( -0.1, -0.5) ( -0.1, -0.5) ( 0.1, 0.7)
in-place inverse FFT of Y:
Y =
(0.1, 0.2) (0.7, 0.8) ( 1.3, 1.4) ( 1.9, 2.0)
(0.3, 0.4) ( 0.9, 1.0) ( 1.5, 1.6) ( 2.1, 2.2)
(0.5, 0.6) ( 1.1, 1.2) ( 1.7, 1.8) ( 1.2, 2.0)
```


## Two-Dimensional FFT Routines

For the linear FFT routines, when the input is a two-dimensional array, the FFT is computed along one dimension only, namely, along the columns of the array. The two-dimensional FFT routines take a two-dimensional array as input and compute the FFT along both the column and row dimensions. Specifically, the forward two-dimensional FFT routines compute the following:

$$
X(k, n)=\sum_{l=0}^{N 2-1} \sum_{j=0}^{N l-1} x(j, l) e^{\frac{-2 \pi i l n}{N 2}} e^{\frac{-2 \pi i j k}{N l}}, k=0, \ldots, N 1-1, n=0, \ldots, N 2-1
$$

The inverse two-dimensional FFT routines compute the following:

$$
x(j, l)=\sum_{n=0}^{N 2-1} \sum_{k=0}^{N l-1} X(k, n) e^{\frac{2 \pi i b_{n}}{N 2}} e^{\frac{2 \pi j k}{N 1}}, \quad j=0, \ldots, N 1-1, l=0, \ldots, N 2-1
$$

For both the forward and inverse two-dimensional transforms, a complex-to-complex transform where the input problem is $N 1 \times N 2$ will yield a complex array that is also $N 1 \times N 2$.

When computing a real-to-complex two-dimensional transform (forward FFT), if the real input array is of dimensions $N 1 \times N 2$, the result will be a complex array of dimensions
$\left(\frac{N 1}{2}+1\right) \times N 2$

Conversely, when computing a complex-to-real transform (inverse FFT) of dimensions N1 $\times$
N 2 , an $\left(\frac{N 1}{2}+1\right) \times N 2$ complex array is required as input. As with the real-to-complex and complex-to-real linear FFT, because of conjugate symmetry, only the first ${ }^{\frac{N I}{2}+1}$ complex data points need to be stored in the input or output array along the first dimension. The complex
subarray $X\left(\frac{N 1}{2}+1: N 1-1,:\right) \quad$ can be obtained from $X\left(0: \frac{N 1}{2},:\right)$ as follows:

$$
\begin{aligned}
X(k, n) & =X *(N 1-k, n) \\
k & =\frac{N 1}{2}+1, \ldots, N 1-1 \\
n & =0, \ldots, N 2-1
\end{aligned}
$$

To compute a two-dimensional transform, an FFT routine must be called twice. One call initializes the routine and the second call actually computes the transform. The initialization includes computing the factors of $N 1$ and $N 2$ and the trigonometric weights associated with those factors. In subsequent forward or inverse transforms, initialization is not necessary as long as $N 1$ and $N 2$ remain unchanged.

IMPORTANT: Upon returning from a two-dimensional FFT routine, $Y(0: N-1,:)$ contains the transform results and the original contents of $Y(N:$ LDY-1, :) is overwritten. Here, $N=N 1$ in the complex-to-complex and complex-to-real transforms and $N=\frac{N I}{2}+1$ in the real-to-complex transform.

The following table Table 13, "Single Precision Two-Dimensional FFT Routines," on page 90 lists the single precision two-dimensional FFT routines and their purposes. The same information applies to the corresponding double precision routines except that their data types are double precision and double complex. See Table 13, "Single Precision Two-Dimensional FFT Routines," on page 90 for the mapping. Refer to the individual man pages for a complete description of the routines and their arguments.

TABLE 13 Single Precision Two-Dimensional FFT Routines

| Name | Purpose | Size, Type of Input | Size, Type of Output | Leading Dimension Requirements |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SFFTC2 | OPT $=0$ initialization |  |  |  |  |
|  | OPT $=-1$ real-to-complex forward two-dimensional FFT | N1 $\times$ N2, Real | $\left(\frac{N 1}{2}+1\right) \times N 2$ | $\begin{aligned} & \operatorname{LDX1}=2 \times \text { LDY } 1 \\ & \text { LDY1 } \geq \frac{N 1}{2}+1 \end{aligned}$ | $\begin{aligned} & \operatorname{LDX1} \geq N 1 \\ & \text { LDY1 } \geq \frac{N I}{2}+1 \end{aligned}$ |
|  |  |  | Complex |  |  |
| CFFTS2 | OPT $=0$ initialization |  |  |  |  |
|  | OPT = 1 complex-to-real inverse two-dimensional FFT | $\left(\frac{N 1}{2}+1\right) \times N 2$ | $N 1 \times N 2$, Real | $\operatorname{LDX} 1 \geq \frac{N I}{2}+1$ | $\operatorname{LDX} 1 \geq \frac{N I}{2}+1$ |
|  |  |  |  | LDY1 $=2 \times$ LDX1 | LDY1 $\geq 2 \times$ LDX1 |
|  |  | Complex |  |  | LDY1 is even |
| CFFTC2 | OPT $=0$ initialization |  |  |  |  |
|  | OPT = -1 complex-to-complex forward two-dimensional FFT | $N 1 \times N 2$, Complex | $N 1 \times N 2$, Complex | LDX1 $\geq$ N1 | LDX1 $\geq$ N1 |
|  |  |  |  | LDY1 $=$ LDX1 | LDY1 $\geq$ N1 |
|  | OPT = 1 complex-to-complex inverse two-dimensional FFT | $N 1 \times N 2$, Complex | $N 1 \times N 2$, Complex | LDX1 $\geq$ N1 | LDX1 $\geq$ N1 |
|  |  |  |  | LDY1 = LDX1 | LDY1 = LDX1 |

Note -Note the following about the table Table 13, "Single Precision Two-Dimensional FFT Routines," on page 90

- LDX1 is leading dimension of input array.
- LDY1 is leading dimension of output array.
- N 1 is first dimension of the FFT problem.
- N2 is second dimension of the FFT problem.
- When calling routines with OPT $=0$ to initialize the routine, the only error checking that is done is to determine if $N 1, N 2<0$.

EXAMPLE 12 Two-Dimensional Real-to-Complex FFT and Complex-to-Real FFT of a TwoDimensional Array

The following example shows how to compute a two-dimensional real-to-complex FFT and complex-to-real FFT of a two-dimensional array.

```
my_system% cat testsc2.f
    PROGRAM TESTSC2
    IMPLICIT NONE
    INTEGER, PARAMETER :: N1 = 3, N2 = 4, LDX1 = N1,
    $ LDY1 = N1/2+1, LDR1 = 2*(N1/2+1)
    INTEGER LW, IERR, I, J, K, IFAC(128*2)
    REAL, PARAMETER :: ONE = 1.0, SCALE = ONE/(N1*N2)
    REAL :: V(LDR1,N2), X(LDX1, N2), Z(LDR1,N2),
    $ SW(2*N2), TRIGS(2*(N1+N2))
    COMPLEX :: Y(LDY1,N2)
    WRITE(*,*) $'Two-dimensional complex-to-real and real-to-complex FFT'
    WRITE(*,*)
    X = RESHAPE(SOURCE = (/.1, .2, .3, .4, .5, .6, .7, .8,
    $ 2.0,1.0, 1.1, 1.2/), SHAPE=(/LDX1,N2/))
    DO I = 1, N2
        V(1:N1,I) = X(1:N1,I)
    END DO
    WRITE(*,*) 'X ='
    DO I = 1, N1
            WRITE(*,'(5(F5.1,2X))') (X(I,J), J = 1, N2)
    END DO
    WRITE(*,*)
* Initialize trig table and get factors of N1, N2
    CALL SFFTC2(0,N1,N2,ONE,X,LDX1,Y,LDY1,TRIGS,
    $
        IFAC,SW,0,IERR)
* Compute 2-dimensional out-of-place forward FFT.
* Let FFT routine allocate memory.
* cannot do an in-place transform in X because LDX1 < 2*(N1/2+1)
    CALL SFFTC2(-1,N1,N2,ONE,X,LDX1,Y,LDY1,TRIGS,
    $ IFAC,SW,0,IERR)
    WRITE(*,*) 'out-of-place forward FFT of X:'
    WRITE(*,*)'Y ='
    DO I = 1, N1/2+1
        WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))')('(',REAL(Y(I,J)),
    $ ',',AIMAG(Y(I,J)),')', J = 1, N2)
    END DO
    WRITE(*,*)
* Compute 2-dimensional in-place forward FFT.
* Use workspace already allocated.
* V which is real array containing input data is also
* used to store complex results; as a complex array, its first
* leading dimension is LDR1/2.
```

```
    CALL SFFTC2(-1,N1,N2,ONE,V,LDR1,V,LDR1/2,TRIGS,
    $ IFAC,SW,LW,IERR)
    WRITE(*,*) 'in-place forward FFT of X:'
    CALL PRINT_REAL_AS_COMPLEX(N1/2+1, N2, 1, V, LDR1/2, N2)
* Compute 2-dimensional out-of-place inverse FFT.
* Leading dimension of Z must be even
    CALL CFFTS2(1,N1,N2,SCALE,Y,LDY1,Z,LDR1,TRIGS,
        $
                        IFAC,SW,0,IERR)
    WRITE(*,*) 'out-of-place inverse FFT of Y:'
    DO I = 1, N1
        WRITE(*,'(5(F5.1,2X))') (Z(I,J), J = 1, N2)
    END DO
    WRITE(*,*)
* Compute 2-dimensional in-place inverse FFT.
* Y which is complex array containing input data is also
* used to store real results; as a real array, its first
* leading dimension is 2*LDY1.
    CALL CFFTS2(1,N1,N2,SCALE,Y,LDY1,Y,2*LDY1,
        $
                    TRIGS,IFAC,SW, 0,IERR)
    WRITE(*,*) 'in-place inverse FFT of Y:'
    CALL PRINT_COMPLEX_AS_REAL(N1, N2, 1, Y, 2*LDY1, N2)
    END PROGRAM TESTSC2
    SUBROUTINE PRINT_COMPLEX_AS_REAL(N1, N2, N3, A, LD1, LD2)
    INTEGER N1, N2, N3, I, J, K
    REAL A(LD1, LD2, *)
    DO K = 1, N3
        DO I = 1, N1
            WRITE(*,'(5(F5.1,2X))') (A(I,J,K), J = 1, N2)
        END DO
        WRITE(*,*)
    END DO
    END
    SUBROUTINE PRINT_REAL_AS_COMPLEX(N1, N2, N3, A, LD1, LD2)
    INTEGER N1, N2, N3, I, J, K
    COMPLEX A(LD1, LD2, *)
    DO K = 1, N3
        DO I = 1, N1
                WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))') ('(',REAL(A(I,J,K)),
    $
                            ',',AIMAG(A(I,J,K)),')', J = 1, N2)
            END DO
            WRITE(*,*)
    END DO
    END
my_system% f95 -dalign testsc2.f -library=sunperf
my_system% a.out
Two-dimensional complex-to-real and real-to-complex FFT
x =
0.1 0.4 0.7 1.0
```

```
0.2 0.5 0.8 1.1
0.3 0.6 2.0 1.2
out-of-place forward FFT of X:
Y =
( 8.9, 0.0) (-2.9, 1.8) (-0.7, 0.0) ( -2.9, -1.8)
(-1.2, 1.3) ( 0.5, -1.0) ( -0.5, 1.0) ( 0.5, -1.0)
in-place forward FFT of X:
( 8.9, 0.0) ( -2.9, 1.8) ( -0.7, 0.0) ( -2.9, -1.8)
(-1.2, 1.3) (0.5, -1.0) ( -0.5, 1.0) ( 0.5, -1.0)
out-of-place inverse FFT of Y:
0.1 0.4 0.7 1.0
0.2 0.5 0.8 1.1
0.3 0.6 2.0 1.2
in-place inverse FFT of Y:
0.1 0.4 0.7 1.0
0.2 0.5 0.8 1.1
0.3 0.6 2.0 1.2
```


## Three-Dimensional FFT Routines

Oracle Developer Studio Performance Library includes routines that compute three-dimensional FFT. In this case, the FFT is computed along all three dimensions of a three-dimensional array. The forward FFT computes

$$
\begin{gathered}
X(k, n, m)=\sum_{h=0}^{N 3-1} \sum_{l=0}^{N 2-1} \sum_{j=0}^{N l-1} x(j, l, h) e^{\frac{-2 \pi i b m}{N 3}} e^{\frac{-2 \pi i b l}{N 2}} e^{\frac{-2 \pi i j}{N l}}, \\
k=0, \ldots, N 1-1 \\
n=0, \ldots, N 2-1 \\
m=0, \ldots, N 3-1
\end{gathered}
$$

and the inverse FFT computes

$$
\begin{gathered}
x(j, l, h)=\sum_{m=0}^{N 3-1} \sum_{n=0}^{N 2-1} \sum_{k=0}^{N l-1} X(k, n, m) e^{\frac{2 \pi i b m}{N 3}} e^{\frac{2 \pi i b n}{N 2}} e^{\frac{2 \pi i k}{N l}}, \\
j=0, \ldots, N 1-1 \\
l=0, \ldots, N 2-1 \\
h=0, \ldots, N 3-1
\end{gathered}
$$

In the complex-to-complex transform, if the input problem is $N 1 \times N 2 \times N 3$, a three-dimensional transform will yield a complex array that is also $N 1 \times N 2 \times N 3$. When computing a real-tocomplex three-dimensional transform, if the real input array is of dimensions $N 1 \times N 2 \times N 3$, the result will be a complex array of dimensions $\left(\frac{N 1}{2}+1\right) \times N 2 \times N 3$. Conversely, when computing a complex-to-real FFT of dimensions $N 1 \times N 2 \times N 3$, an $\left(\frac{N 1}{2}+1\right) \times N 2 \times N 3$ complex array is required as input. As with the real-to-complex and complex-to-real linear FFT, because of conjugate symmetry, only the first ${ }^{\frac{N I}{2}+1}$ complex data points need to be stored along the first dimension. The complex subarray $X\left(\frac{N 1}{2}+1: N 1-1,:,:\right)$ can be obtained from

$$
X\left(0: \frac{N 1}{2}+1, \therefore, \quad:\right) \quad \text { as follows: }
$$

$$
\begin{gathered}
X(k, n, m)=X *(N 1-k, n, m) \\
k=\frac{N 1}{2}+1, \ldots, N 1-1 \\
n=0, \ldots, \quad N 2-1 \\
m=0, \ldots, \quad N 3-1
\end{gathered}
$$

To compute a three-dimensional transform, an FFT routine must be called twice: Once to initialize and once more to actually compute the transform. The initialization includes computing the factors of $N 1, N 2$, and $N 3$ and the trigonometric weights associated with those factors. In subsequent forward or inverse transforms, initialization is not necessary as long as $N 1, N 2$, and $N 3$ remain unchanged.

IMPORTANT: Upon returning from a three-dimensional FFT routine, $Y(0: N-1,:,:$ ) contains the transform results and the original contents of $Y(N: L D Y 1-1,:,:)$ is overwritten. Here, $N=N 1$ in the complex-to-complex and complex-to-real transforms and $N=\frac{N I}{2}+1$ in the real-to-complex transform.

Table 14, "Single Precision Three-Dimensional FFT Routines," on page 95 lists the single precision three-dimensional FFT routines and their purposes. The same information applies to the corresponding double precision routines except that their data types are double precision and double complex. See Table 14, "Single Precision Three-Dimensional FFT Routines," on page 95 for the mapping. See the individual man pages for a complete description of the routines and their arguments.

TABLE 14 Single Precision Three-Dimensional FFT Routines

| Name | Purpose | Size, Type of Input | Size, Type of Output | Leading Dimension Requirements |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SFFTC3 | OPT $=0$ initialization |  |  |  |  |
|  | OPT = - 1 real-tocomplex forward three-dimensional FFT |  | $N 1 \times N 2 \times N 3$, Real | $\left(\frac{N 1}{2}+1\right) \times N 2 \times$ | $\begin{aligned} & \text { LDX1=2 } \times \text { LDY1 } \\ & N 3 \\ & \text { LDXCèm/2lex } \end{aligned}$ | $\begin{aligned} & \operatorname{LDX} 1 \geq N 1 \\ & \operatorname{LDX} 2 \geq N 2 \end{aligned}$ |
|  |  | $\text { LDY1 } \geq \frac{N I}{2}+1$ |  |  | $\text { LDY1 } \geq \frac{N I}{2}+1$ |
|  |  | LDY2 $=$ LDX2 |  |  | LDY2 $\geq$ N2 |
| CFFTS3 | OPT $=0$ initialization |  |  |  |  |
|  | OPT = 1 complex-toreal inverse threedimensional FFT | $\left(\frac{N 1}{2}+1\right) \times N 2 \times$ | $\stackrel{N 1 \times N 2 \times N 3 \text {, Real }}{N 3}$ | $\operatorname{LDX} 1 \geq \frac{N I}{2}+1$ | $\operatorname{LDX1} \geq \frac{N I}{2}+1$ |
|  |  | Complex |  | LDX2 $\geq$ N2 | LDX2 $\geq$ N2 |
|  |  |  |  | LDY1 $=2 \times$ LDX1 | LDY1 $\geq 2 \times$ LDX1 |
|  |  |  |  | LDY2=LDX2 | LDY1 is even |
|  |  |  |  |  | LDY2 $\geq$ N2 |
| CFFTC3 | OPT $=0$ initialization |  |  |  |  |
|  | OPT $=-1$ complex -to-complex forward three-dimensional FFT | $N 1 \times N 2 \times N 3$, Complex | $N 1 \times N 2 \times N 3$, Complex | $\begin{aligned} & \operatorname{LDX} 1 \geq N 1 \\ & \operatorname{LDX} 2 \geq N 2 \end{aligned}$ | $\begin{aligned} & \mathrm{LDX} 1 \geq N 1 \\ & \mathrm{LDX} 2 \geq N 2 \end{aligned}$ |
|  |  |  |  | LDY1=LDX1 | LDY1 $\geq$ N1 |
|  |  |  |  | LDY2=LDX2 | LDY2 $\geq$ N2 |
|  | OPT = 1 complex-tocomplex inverse threedimensional FFT | $N 1 \times N 2 \times N 3$, Complex | $N 1 \times N 2 \times N 3$, Complex | LDX1 $\geq$ N1 LDX2 $\geq$ 22 | LDX1 $\geq$ N1 LDX2 $\geq$ N2 |
|  |  |  |  | LDY1=LDX1 | LDY1 $\geq$ N1 |
|  |  |  |  | LDY2=LDX2 | LDY2 $\geq$ N2 |

Note - Note the following about the table Table 14, "Single Precision Three-Dimensional FFT Routines," on page 95:

- LDX1 is first leading dimension of input array.
- LDX2 is the second leading dimension of the input array.
- LDY1 is the first leading dimension of the output array.
- LDY2 is the second leading dimension of the output array.
- N 1 is the first dimension of the FFT problem.
- N2 is the second dimension of the FFT problem.
- N 3 is the third dimension of the FFT problem.
- When calling routines with $O P T=0$ to initialize the routine, the only error checking that is done is to determine if $N 1, N 2, N 3<0$.

Example 7-4 shows how to compute the three-dimensional real-to-complex FFT and complex-to-real FFT of a three-dimensional array.

EXAMPLE 13 Three-Dimensional Real-to-Complex FFT and Complex-to-Real FFT of a ThreeDimensional Array
my_system\% cat testsc3.f
PROGRAM TESTSC3
IMPLICIT NONE
INTEGER LW, NCPUS, IERR, I, J, K, USING_THREADS, IFAC(128*3)
INTEGER, PARAMETER :: N1 = 3, N2 = 4, N3 = 2, LDX1 = N1,
\$ LDX2 = N2, LDY1 = N1/2+1, LDY2 = N2,
\$ LDR1 $=2^{*}(\mathrm{~N} 1 / 2+1)$, LDR2 $=$ N2
REAL, PARAMETER :: ONE = 1.0, SCALE $=$ ONE/(N1*N2*N3)
REAL :: V(LDR1,LDR2,N3), X(LDX1,LDX2,N3), Z(LDR1,LDR2,N3),
\$
REAL, DIMENSION(:), ALLOCATABLE :: SW
COMPLEX :: Y(LDY1,LDY2,N3)
WRITE (*,*)
\$'Three-dimensional complex-to-real and real-to-complex FFT' WRITE(*,*)

* get number of threads

NCPUS = USING_THREADS()

* compute workspace size required
$\mathrm{LW}=(\operatorname{MAX}(\operatorname{MAX}(\mathrm{N} 1,2 * \mathrm{~N} 2), 2 * \mathrm{~N} 3)+16 * N 3) *$ NCPUS
ALLOCATE(SW(LW))
$\mathrm{X}=$ RESHAPE(SOURCE $=$
\$ (/ .1, .2, .3, .4, .5, .6, .7, .8, .9,1.0,1.1,1.2,
\$ $\quad 4.1,1.2,2.3,3.4,6.5,1.6,2.7,4.8,7.9,1.0,3.1,2.2 /)$,
\$ SHAPE=(/LDX1,LDX2,N3/))

```
    V = RESHAPE(SOURCE =
    $ (/.1,.2,.3,0.,.4,.5,.6,0.,.7,.8,.9,0.,1.0,1.1,1.2,0.,
    $ 4.1,1.2,2.3,0.,3.4,6.5,1.6,0.,2.7,4.8,7.9,0.,
    $ 1.0,3.1,2.2,0./), SHAPE=(/LDR1,LDR2,N3/))
        WRITE(*,*) 'X ='
        DO K = 1, N3
            DO I = 1, N1
                WRITE(*,'(5(F5.1,2X))') (X(I, J,K), J = 1, N2)
            END DO
            WRITE(*,*)
        END DO
* Initialize trig table and get factors of N1, N2 and N3
        CALL SFFTC3(0,N1,N2,N3,ONE,X,LDX1, LDX2,Y,LDY1,LDY2,TRIGS,
        $
        IFAC,SW,0,IERR)
* Compute 3-dimensional out-of-place forward FFT.
* Let FFT routine allocate memory.
* cannot do an in-place transform because LDX1 < 2*(N1/2+1)
    CALL SFFTC3(-1,N1,N2,N3,ONE,X,LDX1,LDX2,Y,LDY1,LDY2,TRIGS,
        $ IFAC,SW,0,IERR)
        WRITE(*,*) 'out-of-place forward FFT of X:'
        WRITE(*,*)'Y ='
        DO K = 1, N3
            DO I = 1, N1/2+1
                WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))')('(',REAL(Y(I, J,K)),
            $
            END DO
            WRITE(*,*)
        END DO
* Compute 3-dimensional in-place forward FFT.
* Use workspace already allocated.
* V which is real array containing input data is also
* used to store complex results; as a complex array, its first
* leading dimension is LDR1/2.
    CALL SFFTC3(-1,N1,N2,N3,ONE,V,LDR1,LDR2,V,LDR1/2,LDR2,TRIGS,
    $
        IFAC,SW,LW, IERR)
    WRITE(*,*) 'in-place forward FFT of X:'
    CALL PRINT_REAL_AS_COMPLEX(N1/2+1, N2, N3, V, LDR1/2, LDR2)
* Compute 3-dimensional out-of-place inverse FFT.
* First leading dimension of Z (LDR1) must be even
    CALL CFFTS3(1,N1,N2,N3,SCALE,Y,LDY1,LDY2,Z,LDR1,LDR2,TRIGS,
    $
    WRITE(*,*) 'out-of-place inverse FFT of Y:'
    DO K = 1, N3
            DO I = 1, N1
                WRITE(*,'(5(F5.1,2X))') (Z(I,J,K), J = 1, N2)
            END DO
            WRITE(*,*)
    END DO
```

```
* Compute 3-dimensional in-place inverse FFT.
* Y which is complex array containing input data is also
* used to store real results; as a real array, its first
* leading dimension is 2*LDY1.
        CALL CFFTS3(1,N1,N2,N3,SCALE,Y,LDY1, LDY2,Y, 2*LDY1,LDY2,
        $ TRIGS,IFAC,SW,LW,IERR)
        WRITE(*,*) 'in-place inverse FFT of Y:'
        CALL PRINT_COMPLEX_AS_REAL(N1, N2, N3, Y, 2*LDY1, LDY2)
        DEALLOCATE(SW)
        END PROGRAM TESTSC3
        SUBROUTINE PRINT_COMPLEX_AS_REAL(N1, N2, N3, A, LD1, LD2)
        INTEGER N1, N2, N3, I, J, K
        REAL A(LD1, LD2, *)
        DO K = 1, N3
            DO I = 1, N1
                WRITE(*,'(5(F5.1,2X))') (A(I,J,K), J = 1, N2)
            END DO
            WRITE(*,*)
        END DO
        END
        SUBROUTINE PRINT_REAL_AS_COMPLEX(N1, N2, N3, A, LD1, LD2)
        INTEGER N1, N2, N3, I, J, K
        COMPLEX A(LD1, LD2), *)
        DO K = 1, N3
            DO I = 1, N1
                WRITE(*,'(5(A1, F5.1,A1,F5.1,A1,2X))') ('(',REAL(A(I,J,K)),
        $
                ',',AIMAG(A(I,J,K)),')', J = 1, N2)
            END DO
            WRITE(*,*)
        END DO
        END
my_system% f95 -dalign testsc3.f -xlibrary=sunperf
my_system% a.out
Three-dimensional complex-to-real and real-to-complex FFT
X =
0.1 0.4 0.7 1.0
0.2 0.5 0.8 1.1
0.3 0.6 0.9 1.2
4.1 3.4 2.7 1.0
1.2 6.5 4.8 3.1
2.3 1.6 7.9 2.2
out-of-place forward FFT of X:
Y =
( 48.6, 0.0) ( -9.6, -3.4) ( 3.4, 0.0) ( -9.6, 3.4)
(-4.2,-1.0) ( 2.5, -2.7) ( 1.0, 8.7) ( 9.5, -0.7)
(-33.0, 0.0) ( 6.0, 7.0) (-7.0, 0.0) ( 6.0, -7.0)
( 3.0, 1.7) (-2.5, 2.7) (-1.0, -8.7) ( -9.5, 0.7)
in-place forward FFT of X:
```

```
(48.6, 0.0) (-9.6, -3.4) ( 3.4, 0.0) ( -9.6, 3.4)
( -4.2, -1.0) ( 2.5, -2.7) ( 1.0, 8.7) ( 9.5, -0.7)
(-33.0, 0.0) ( 6.0, 7.0) ( -7.0, 0.0) ( 6.0, -7.0)
( 3.0, 1.7) (-2.5, 2.7) ( -1.0, -8.7) ( -9.5, 0.7)
out-of-place inverse FFT of Y:
0.1 0.4 0.7 1.0
0.2 0.5 0.8 1.1
0.3 0.6 0.9 1.2
4.1 3.4 2.7 1.0
1.2 6.5 4.8 3.1
2.3 1.6 7.9 2.2
in-place inverse FFT of Y:
0.1 0.4 0.7 1.0
0.2 0.5 0.8 1.1
0.3 0.6 0.9 1.2
4.1 3.4 2.7 1.0
1.2 6.5 4.8 3.1
2.3 1.6 7.9 2.2
```


## Comments

When doing an in-place real-to-complex or complex-to-real transform, care must be taken to ensure the size of the input array is large enough to hold the results. For example, if the input is of type complex stored in a complex array with first leading dimension $N$, then to use the same array to store the real results, its first leading dimension as a real output array would be $2 \times N$. Conversely, if the input is of type real stored in a real array with first leading dimension $2 \times N$, then to use the same array to store the complex results, its first leading dimension as a complex output array would be $N$. Leading dimension requirements for in-place and out-ofplace transforms can be found in Table 12, "Single Precision Linear FFT Routines," on page 81, Table 13, "Single Precision Two-Dimensional FFT Routines," on page 90, and Table 14, "Single Precision Three-Dimensional FFT Routines," on page 95.

In the linear and multi-dimensional FFT, the transform between real and complex data through a real-to-complex or complex-to-real transform can be confusing because $N 1$ real data points correspond to ${ }^{\frac{N I}{2}+1}$ complex data points. $N 1$ real data points do map to $N 1$ complex data points, but because there is conjugate symmetry in the complex data, only ${ }^{\frac{N I}{2}+1}$ data points need to be stored as input in the complex-to-real transform and as output in the real-to-complex transform. In the multi-dimensional FFT, symmetry exists along all the dimensions, not just in the first. However, the two-dimensional and three-dimensional FFT routines store the complex data of the second and third dimensions in their entirety.

While the FFT routines accept any size of $N 1, N 2$ and $N 3$, FFTs can be computed most efficiently when values of $N 1, N 2$ and $N 3$ can be decomposed into relatively small primes. A real-to-complex or a complex-to-real transform can be computed most efficiently when
$N 1, N 2, N 3=2^{p} \times 3^{q} \times 4^{r} \times 5^{s}$,
and a complex-to-complex transform can be computed most efficiently when

$$
N 1, N 2, N 3=2^{p} \times 3^{q} \times 4^{r} \times 5^{s} \times 7^{t} \times 11^{u} \times 13^{v}
$$

where $p, q, r, s, t, u$, and $v$ are integers and $p, q, r, s, t, u, v \geq 0$.
The function $x$ FFTOPT can be used to determine the optimal sequence length, as shown in Example 14, " RFFTOPT Example," on page 100. Given an input sequence length, the function returns an optimal length that is closest in size to the original length.

EXAMPLE 14 RFFTOPT Example

```
my_system% cat fft_ex01.f
    PROGRAM TEST
    INTEGER N, N1, N2, N3, RFFTOPT
C
    N = 1024
    N1 = 1019
    N2 = 71
    N3 = 49
C
    PRINT *, 'N Original N Suggested'
    PRINT '(I5, I12)', (N, RFFTOPT(N))
    PRINT '(I5, I12', (N1, RFFTOPT(N1))
    PRINT '(I5, I12)', (N2, RFFTOPT(N2))
    PRINT '(I5, I12)', (N3, RFFTOPT(N3))
    END
```

my_system\% f95 -dalign fft_ex01.f -library=sunperf
my_system\% a.out
N Original N Suggested
10241024
$1019 \quad 1024$
$71 \quad 72$
$49 \quad 49$

## Cosine and Sine Transforms

Input to the DFT that possess special symmetries occur in various applications. A transform that exploits symmetry usually saves in storage and computational count, such as with the real-to-complex and complex-to-real FFT transforms. The Performance Library cosine and sine transforms are special cases of FFT routines that take advantage of the symmetry properties found in even and odd functions.

Note - Oracle Developer Studio Performance Library sine and cosine transform routines are based on the routines contained in FFTPACK (http://www. netlib. org/fftpack/). Routines with a V prefix are vectorized routines that are based on the routines contained in VFFTPACK (http://www.netlib.org/vfftpack/).

## Fast Cosine and Sine Transform Routines

The following tables list the Oracle Developer Studio Performance Library fast cosine and sine transforms. Names of double precision routines are in square brackets. Routines whose name begins with 'V' can compute the transform of one or more sequences simultaneously. Those whose name ends with 'I' are initialization routines.

- Table 15, "Fast Cosine Transforms for Even Sequences Routines and Their Arguments," on page 101
- Table 16, "Fast Cosine Transforms for Quarter-Wave Even Sequences Routines and Their Arguments," on page 102
- Table 17, "Fast Sine Transforms for Odd Sequences Routines and Their Arguments," on page 102
- Table 18, "Fast Sine Transforms for Quarter-Wave Odd Sequences Routines and Their Arguments," on page 102

TABLE 15 Fast Cosine Transforms for Even Sequences Routines and Their Arguments

| Routine Name | Arguments |
| :--- | :--- |
| COST [DCOST] | $($ LEN +1, X, WORK $)$ |
| COSTI [DCOSTI] | (LEN+1, WORK) |
| VCOST [VDCOST] | $(M$, LEN +1, X, WORK, LD, TABLE $)$ |
| VCOSTI [VDCOSTI] | (LEN+1, TABLE) |

TABLE 16 Fast Cosine Transforms for Quarter-Wave Even Sequences Routines and Their Arguments

| Routine Name | Arguments |
| :--- | :--- |
| COSQF [DCOSQF] | $($ LEN, X, WORK) |
| COSQB [DCOSQB] | $($ LEN, $X$, WORK $)$ |
| COSQI [DCOSQI] | $($ LEN, WORK $)$ |
| VCOSQF [VDCOSQF] | $(M$, LEN, X, WORK, LD, TABLE $)$ |
| VCOSQB [VDCOSQB] | $(M$, LEN, X, WORK, LD, TABLE $)$ |
| VCOSQI [VDCOSQI] | $(L E N$, TABLE) |

TABLE 17 Fast Sine Transforms for Odd Sequences Routines and Their Arguments

| Routine Name | Arguments |
| :--- | :--- |
| SINT [DSINT] | (LEN-1, X, WORK) |
| SINTI [DSINTI] | (LEN-1, WORK) |
| VSINT [VDSINT] | (M, LEN-1, X, WORK, LD, TABLE) |
| VSINTI [VDSINTI] | (LEN-1, TABLE) |

TABLE 18 Fast Sine Transforms for Quarter-Wave Odd Sequences Routines and Their Arguments

| Routine Name | Arguments |
| :--- | :--- |
| SINQF [DSINQF] | (LEN, X, WORK) |
| SINQB [DSINQB] | (LEN, X, WORK) |
| SINQI [DSINQI] | (LEN, WORK) |
| VSINQF [VDSINQF] | $(M$, LEN, X, WORK, LD, TABLE) |
| VSINQB [VDSINQB] | $(M$, LEN, X, WORK, LD, TABLE) |
| VSINQI [VDSINQI] | (LEN, TABLE) Notes: |

Note - Note the following information about the previous tables:

- M : Number of sequences to be transformed.
- LEN, LEN-1, LEN+1: Length of the input sequence or sequences.
- X : A real array which contains the sequence or sequences to be transformed. On output, the real transform results are stored in X .
- TABLE: Array of constants particular to a transform size that is required by the transform routine. The constants are computed by the initialization routine.
- WORK: Workspace required by the transform routine. In routines that operate on a single sequence, WORK also contains constants computed by the initialization routine.


## Fast Sine Transforms

Another type of symmetry that is commonly encountered is the odd symmetry where $x(n)$ $=-x(-n)$ for $n=-N+1, \ldots, 0, \ldots, N$. As in the case of the fast cosine transform, the fast sine transform (FST) takes advantage of the odd symmetry to save memory and computation. For a real odd sequence $x$, symmetry implies that $x(0)=-x(0)=0$. Therefore, if $x$ is of length $2 N$ then only $N=1$ values of $x$ are required to compute the FST. Routine SINT computes the FST of a single real odd sequence while VSINT computes the FST of one or more sequences. Before calling [V]SINT, [V]SINTI must be called to compute trigonometric constants and factors associated with input length $N-1$. The FST is its own inverse transform. Calling VSINT twice will result in the original $N-1$ data points. Calling SINT twice will result in the original $N-1$ data points multiplied by $2 N$.

An odd sequence with symmetry such that $x(n)=-x(-n-1)$, where $-N+1, \ldots, 0, \ldots, N$ is said to have quarter-wave odd symmetry. SINQF and SINQB compute the FST and its inverse, respectively, of a single real quarter-wave odd sequence while VSINQF and VSINQB operate on one or more sequences. SINQB is unnormalized, so using the results of SINQF as input in SINQB produces the original sequence scaled by a factor of $4 N$. However, VSINQB is normalized, so a call to VSINQF followed by a call to VSINQB will produce the original sequence. An FST of a real sequence of length $2 N$ that has quarter-wave odd symmetry requires $N$ input data points and produces an $N$-point resulting sequence. Initialization is required before calling the transform routines by calling [V]SINQI.

## Fast Cosine Transforms

A special form of the FFT that operates on real even sequences is the fast cosine transform (FCT). A real sequence $x$ is said to have even symmetry if $x(n)=x(-n)$ where $n=-N+1, \ldots$,
$0, \ldots, N$. An FCT of a sequence of length $2 N$ requires $N+1$ input data points and produces a sequence of size $N+1$. Routine COST computes the FCT of a single real even sequence while VCOST computes the FCT of one or more sequences. Before calling [V]COST, [V]COSTI must be called to compute trigonometric constants and factors associated with input length $N+1$. The FCT is its own inverse transform. Calling VCOST twice will result in the original $N+1$ data points. Calling COST twice will result in the original $N+1$ data points multiplied by $2 N$.

An even sequence $x$ with symmetry such that $x(n)=x(-n-1)$ where $n=-N+1, \ldots, 0, \ldots$, $N$ is said to have quarter-wave even symmetry. COSQF and COSQB compute the FCT and its inverse, respectively, of a single real quarter-wave even sequence. VCOSQF and VCOSQB operate on one or more sequences. The results of [V]COSQB are unnormalized, and if scaled by $\frac{1}{4 \mathrm{~N}}$ the original sequences are obtained. An FCT of a real sequence of length $2 N$ that has quarterwave even symmetry requires $N$ input data points and produces an $N$-point resulting sequence. Initialization is required before calling the transform routines by calling [V]COSQI.

## Discrete Fast Cosine and Sine Transforms and Their Inverse

Oracle Developer Studio Performance Library routines use the equations in the following sections to compute the fast cosine and sine transforms and inverse transforms.

## [D] COST: Forward and Inverse Fast Cosine Transform (FCT) of a Sequence

The forward and inverse FCT of a sequence is computed as:

$$
X(k)=x(0)+2 \sum_{n=1}^{N-1} x(n) \cos \left(\frac{\pi n k}{N}\right)+x(N) \cos (\pi k), k=0, \ldots, N
$$

[D]COST Notes:

- $N+1$ values are needed to compute the FCT of an $N$-point sequence.
- [D]COST also computes the inverse transform. When [D]COST is called twice, the result will be the original sequence scaled by $\frac{1}{2 N}$


## V[D]COST: Forward and Inverse Fast Cosine Transforms of Multiple Sequences (VFCT)

The forward and inverse FCTs of multiple sequences are computed as:
For $i=0, M-1$

$$
X(i, k)=\frac{x(i, 0)}{2 \mathrm{~N}}+\frac{1}{N} \sum_{n=1}^{\mathrm{N}-1} x(i, n) \cos \left(\frac{\pi n k}{N}\right)+x \frac{(i, N)}{2 \mathrm{~N}} \cos (\pi k), k=0, \ldots, N
$$

## V[D]COST Notes

- $M \times(N+1)$ values are needed to compute the VFCT of $M N$-point sequences.
- The input and output sequences are stored row-wise.
- V [D] COST is normalized and is its own inverse. When V[D]COST is called twice, the result will be the original data.


## [D] COSQF: Forward FCT of a Quarter-Wave Even Sequence

The forward FCT of a quarter-wave even sequence is computed as:

$$
X(k)=x(0)+2 \sum_{n=1}^{N-1} x(n) \cos \left(\frac{\pi(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right), k=0, \ldots, N-1
$$

$N$ values are needed to compute the forward FCT of an $N$-point quarter-wave even sequence.

## [D]COSQB: Inverse FCT of a Quarter-Wave Even Sequence

The inverse FCT of a quarter-wave even sequence is computed as

$$
x(n)=\sum_{k=0}^{N-1} X(k) \cos \left(\frac{\pi n(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right), n=0, \ldots, N-1
$$

Calling the forward and inverse routines will result in the original input scaled by $\frac{1}{4 \mathrm{~N}}$.

## v[D]COSQF: Forward FCT of One or More Quarter-Wave Even Sequences

The forward FCT of one or more quarter-wave even sequences is computed as
For $i=0, M-1$

$$
X(i, k)=\frac{1}{N}\left[x(i, 0)+2 \sum_{n=1}^{N-1} x(i, n) \cos \left(\frac{\pi n(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right)\right], k=0, \ldots, N-1
$$

## V[D]COSQF Notes:

- The input and output sequences are stored row-wise.
- The transform is normalized so that if the inverse routine V[D]COSQB is called immediately after calling $\vee[\mathrm{D}] \operatorname{COSQF}$, the original data is obtained.


## v[D]COSQB: Inverse FCT of One or More Quarter-Wave Even Sequences

The inverse FCT of one or more quarter-wave even sequences is computed as
For $i=0, M-1$

$$
x(i, n)=\sum_{k=0}^{N-1} X(i, k) \cos \left(\frac{\pi n(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right), n=0, \ldots, N-1
$$

## V[D]COSQB Notes:

- The input and output sequences are stored row-wise.
- The transform is normalized so that if V [D]COSQB is called immediately after calling V [D] COSQF, the original data is obtained.


## [D]SINT: Forward and Inverse Fast Sine Transform (FST) of a Sequence

The forward and inverse FST of a sequence is computed as

$$
X(k)=2 \sum_{n=0}^{N-2} x(n) \sin \left(\frac{\pi(n+1)(k+1)}{N}\right), k=0, \ldots, N-2
$$

[D]SINT Notes:

- $N-1$ values are needed to compute the FST of an $N$-point sequence.
- [D]SINT also computes the inverse transform. When [D]SINT is called twice, the result will be the original sequence scaled by $\frac{1}{2 N}$.


## V[D]SINT: Forward and Inverse Fast Sine Transforms of Multiple Sequences (VFST)

The forward and inverse fast sine transforms of multiple sequences are computed as
For $i=0, M-1$

$$
X(i, k)=\frac{2}{\sqrt{2 \mathrm{~N}}} \sum_{n=0}^{N-2} x(i, n) \sin \left(\frac{\pi(n+1)(k+1)}{N}\right), k=0, \ldots, N-2 .
$$

V[D]SINT Notes:

- $M \times(N-1)$ values are needed to compute the VFST of $M N$-point sequences.
- The input and output sequences are stored row-wise.
- V[D]SINT is normalized and is its own inverse. Calling V[D]SINT twice yields the original data.


## [D]SINQF: Forward FST of a Quarter-Wave Odd Sequence

The forward FST of a quarter-wave odd sequence is computed as

$$
X(k)=2 \sum_{n=0}^{N-2} x(n) \sin \left(\frac{\pi(n+1)(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right)+x(N-1) \cos (\pi k), \quad k=0, \ldots, N-1
$$

$N$ values are needed to compute the forward FST of an $N$-point quarter-wave odd sequence.

## [D]SINQB: Inverse FST of a Quarter-Wave Odd Sequence

The inverse FST of a quarter-wave odd sequence is computed as

$$
x(n)=2 \sum_{k=0}^{N-1} X(k) \sin \left(\frac{\pi(n+1)(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right), n=0, \ldots, N-1
$$

Calling the forward and inverse routines will result in the original input scaled by $\frac{1}{4 \mathrm{~N}}$.

## v[D]SINQF: Forward FST of One or More Quarter-Wave Odd Sequences

The forward FST of one or more quarter-wave odd sequences is computed as
For $i=0, M-1$

$$
X(i, k)=\frac{1}{\sqrt{4 \mathrm{~N}}}\left[2 \sum_{n=0}^{N-2} x(n, i) \sin \left(\frac{\pi(n+1)(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right)+x(N-1, i) \cos \pi k\right], k=0, \ldots, N-1
$$

V[D]SINQF Notes:

- The input and output sequences are stored row-wise.
- The transform is normalized so that if the inverse routine V[D]SINQB is called immediately after calling V[D]SINQF, the original data is obtained.


## V[D]SINQB: Inverse FST of One or More Quarter-Wave Odd Sequences

The inverse FST of one or more quarter-wave odd sequences is computed as
For $i=0, M-1$

$$
x(n, i)=\frac{4}{\sqrt{4 \mathrm{~N}}} \sum_{k=0}^{N-1} X(k, i) \sin \left(\frac{\pi(n+1)(2 \mathrm{k}+1)}{2 \mathrm{~N}}\right), \quad n=0, \ldots, N-1
$$

V[D]SINQB Notes:

- The input and output sequences are stored row-wise.
- The transform is normalized, so that if V[D]SINQB is called immediately after calling V[D] SINQF, the original data is obtained.


## Fast Cosine Transform Examples

Example 15, "Computer FCT and Inverse FCT of Single Real Even Sequence," on page 109 calls COST to compute the FCT and the inverse transform of a real even sequence. If the real sequence is of length $2 N$, only $N+1$ input data points need to be stored and the number of resulting data points is also $N+1$. The results are stored in the input array.

## EXAMPLE 15 Computer FCT and Inverse FCT of Single Real Even Sequence

```
my_system% cat cost.f
    program Drive cost
    implicit none
    integer,parameter :: len=4
    real x(0:len),work(3*(len+1)+15), z(0:len), scale
    integer i
    scale = 1.0/(2.0*len)
    call RANDOM_NUMBER(x(0:len))
    z(0:len) = x(0:len)
        write(*,'(a25,i1,a10,i1,a12)')'Input sequence of length ',
    $ len,' requires ', len+1,' data points'
    write(*,'(5(f8.3,2x),/)')(x(i),i=0,len)
    call costi(len+1, work)
    call cost(len+1, z, work)
    write(*,*)'Forward fast cosine transform'
    write(*,'(5(f8.3,2x),/)')(z(i),i=0,len)
    call cost(len+1, z, work)
    write(*,*)
    $ 'Inverse fast cosine transform (results scaled by 1/2*N)'
        write(*,'(5(f8.3,2x),/)')(z(i)*scale,i=0,len)
        end
my_system% f95 -dalign cost.f -library=sunperf
my_system% a.out
Input sequence of length 4 requires 5 data points
0.557 0.603 0.210 0.352 0.867
Forward fast cosine transform
3.753 0.046 1.004-0.666-0.066
Inverse fast cosine transform (results scaled by 1/2*N)
0.557 0.603 0.210 0.352 0.867
```

Example 16, "Compute the FCT and the Inverse FCT of Two Real Quarter-wave Even Sequences," on page 110 calls VCOSQF and VCOSQB to compute the FCT and the inverse FCT,
respectively, of two real quarter-wave even sequences. If the real sequences are of length 2 N , only $N$ input data points need to be stored, and the number of resulting data points is also $N$. The results are stored in the input array.

EXAMPLE 16 Compute the FCT and the Inverse FCT of Two Real Quarter-wave Even Sequences

```
my_system% cat vcosq.f
    program vcosq
    implicit none
    integer,parameter :: len=4, m = 2, ld = m+1
    real x(ld,len),xt(ld,len),work(3*len+15), z(ld,len)
    integer i, j
    call RANDOM_NUMBER(x)
    z = x
    write(*,'(a27,il)')' Input sequences of length ',len
    do j = 1,m
        write(*,'(a3,i1,a4,4(f5.3,2x),a1,/)')
    $ 'seq',j,' = (',(x(j,i),i=1,len),')'
    end do
    call vcosqi(len, work)
    call vcosqf(m,len, z, xt, ld, work)
    write(*,*)
    $ 'Forward fast cosine transform for quarter-wave even sequences'
    do j = 1,m
        write(*,'(a3,i1,a4,4(f5.3,2x),a1,/)')
    $ 'seq',j,' = (',(z(j,i),i=1,len),')'
    end do
    call vcosqb(m,len, z, xt, ld, work)
    write(*,*)
$ 'Inverse fast cosine transform for quarter-wave even sequences'
    write(*,*)'(results are normalized)'
    do j = 1,m
        write(*,'(a3,i1,a4,4(f5.3,2x),a1,/)')
    $ 'seq',j,' = (',(z(j,i),i=1,len),')'
    end do
    end
my_system% f95 -dalign vcosq.f -library=sunperf
my_system% a.out
Input sequences of length 4
seq1 = (0.557 0.352 0.990 0.539 )
seq2 = (0.603 0.867 0.417 0.156 )
Forward fast cosine transform for quarter-wave even sequences
seq1 = (0.755 -. 392 -.029 0.224 )
seq2 = (0.729 0.097 -.091 -. 132 )
Inverse fast cosine transform for quarter-wave even sequences
(results are normalized)
```

```
seq1 = (0.557 0.352 0.990 0.539 )
seq2 = (0.603 0.867 0.417 0.156 )
```


## Fast Sine Transform Examples

In the following example Example 17, "Compute the FCT and the Inverse FCT of Two Real Quarter-wave Even Sequences," on page 111, SINT is called to compute the FST and the inverse transform of a real odd sequence. If the real sequence is of length $2 N$, only $N-1$ input data points need to be stored and the number of resulting data points is also $N-1$. The results are stored in the input array.

EXAMPLE 17 Compute the FCT and the Inverse FCT of Two Real Quarter-wave Even Sequences

```
my_system% cat sint.f
    program Drive sint
    implicit none
    integer,parameter :: len=4
    real x(0:len-2),work(3*(len-1)+15), z(0:len-2), scale
    integer i
    call RANDOM_NUMBER(x(0:len-2))
    z(0:len-2) = x(0:len-2)
    scale = 1.0/(2.0*len)
    write(*,'(a25,i1,a10,i1,a12)')'Input sequence of length ',
    $ len,' requires ', len-1,' data points'
        write(*,'(3(f8.3,2x),/)')(x(i),i=0,len-2)
        call sinti(len-1, work)
        call sint(len-1, z, work)
        write(*,*)'Forward fast sine transform'
        write(*,'(3(f8.3,2x),/)')(z(i),i=0,len-2)
        call sint(len-1, z, work)
        write(*,*)
    $ 'Inverse fast sine transform (results scaled by 1/2*N)'
        write(*,'(3(f8.3,2x),/)')(z(i)*scale,i=0,len-2)
        end
my_system% f95 -dalign sint.f -library=sunperf
my_system% a.out
Input sequence of length 4 requires 3 data points
0.557 0.603 0.210
Forward fast sine transform
2.291 0.694-0.122
Inverse fast sine transform (results scaled by 1/2*N)
0.557 0.603 0.210
```

In the following example Example 18, "Compute FST and Inverse FST of Two Real QuarterWave Odd Sequences," on page 112, VSINQF and VSINQB are called to compute the FST and
inverse FST, respectively, of two real quarter-wave odd sequences. If the real sequence is of length $2 N$, only $N$ input data points need to be stored and the number of resulting data points is also $N$. The results are stored in the input array.

EXAMPLE 18 Compute FST and Inverse FST of Two Real Quarter-Wave Odd Sequences

```
my system% cat vsinq.f
        program vsinq
        implicit none
        integer,parameter :: len=4, m = 2, ld = m+1
        real x(ld,len),xt(ld,len),work(3*len+15), z(ld,len)
        integer i, j
        call RANDOM_NUMBER(x)
        z = x
        write(*,'(a27,i1)')' Input sequences of length ',len
        do j = 1,m
            write(*,'(a3,i1,a4,4(f5.3,2x),a1,/)')
    $ 'seq',j,' = (',(x(j,i),i=1,len),')'
        end do
        call vsinqi(len, work)
        call vsinqf(m,len, z, xt, ld, work)
        write(*,*)
    $ 'Forward fast sine transform for quarter-wave odd sequences'
        do j = 1,m
        write(*,'(a3,i1,a4,4(f5.3,2x),a1,/)')
    $ 'seq',j,' = (',(z(j,i),i=1,len),')'
        end do
        call vsinqb(m,len, z, xt, ld, work)
        write(*,*)
    $ 'Inverse fast sine transform for quarter-wave odd sequences'
        write(*,*)'(results are normalized)'
        do j = 1,m
        write(*,'(a3,i1,a4,4(f5.3,2x),a1,/)')
    $ 'seq',j,' = (',(z(j,i),i=1,len),')'
        end do
        end
my_system% f95 vsinq.f -library=sunperf
my_system% a.out
Input sequences of length 4
seq1 = (0.557 0.352 0.990 0.539 )
seq2 = (0.603 0.867 0.417 0.156 )
Forward fast sine transform for quarter-wave odd sequences
seq1 = (0.823 0.057 0.078 0.305 )
seq2 = (0.654 0.466-.069 -.037 )
Inverse fast sine transform for quarter-wave odd sequences
(results are normalized)
seq1 = (0.557 0.352 0.990 0.539 )
```

```
seq2 = (0.603 0.867 0.417 0.156)
```


## Convolution and Correlation

Two applications of the FFT that are frequently encountered especially in the signal processing area are the discrete convolution and discrete correlation operations.

## Convolution Operation

Given two functions $x(t)$ and $y(t)$, the Fourier transform of the convolution of $x(t)$ and $y(t)$, denoted as $\mathrm{x} * \mathrm{y}$, is the product of their individual Fourier transforms: DFT ( $\mathrm{x} * \mathrm{y}$ ) $=X(\cdot) Y$ where * denotes the convolution operation and $(\bullet)$ denotes pointwise multiplication.

Typically, $x(t)$ is a continuous and periodic signal that is represented discretely by a set of N data points $x_{\mathrm{j}}, j=0, \ldots, N-1$, sampled over a finite duration, usually for one period of $x(t)$ at equal intervals. $y(t)$ is usually a response that starts out as zero, peaks to a maximum value, and then returns to zero. Discretizing $y(t)$ at equal intervals produces a set of $N$ data points, $y_{\mathrm{k}}, k=$ $0, \ldots, N-1$. If the actual number of samplings in $y_{\mathrm{k}}$ is less than $N$, the data can be padded with zeros. The discrete convolution can then be defined as
$(x * y)_{j} \equiv \sum_{k=\frac{-N}{2}+1}^{\frac{N}{2}} x_{j-k} y_{k}, j=0, \ldots, N-1$

The values of $y_{k}, k=-\frac{N}{2}+1, \ldots, \frac{N}{2}$
are the same as those of $k=0, \ldots, N-1$ but in the wrap-around order.

The Oracle Developer Studio Performance Library routines enable you to compute the convolution by using the definition above with $k=0, \ldots, N-1$, or by using the FFT. If the FFT is used to compute the convolution of two sequences, the following steps are performed:

- Compute $X=$ forward FFT of x
- Compute $Y=$ forward FFT of y
- Compute $Z=X(\cdot) Y<=>\operatorname{DFT}(x * y)$
- Compute $z=$ inverse FFT of $Z ; z=\left(x^{*} y\right)$

One interesting characteristic of convolution is that the product of two polynomials is actually a convolution. A product of an $m$-term polynomial

$$
a(x)=a_{0}+a_{1} x+\ldots+a_{m-1} \mathrm{x}^{m-1}
$$

and an $n$-term polynomial

$$
b(x)=b_{0}+b_{1} x+\ldots+b_{n-1} x^{n-1}
$$

has $m+n-1$ coefficients that can be obtained by:

$$
c_{k}=\sum_{j=\max ((k-(m-1)), 0)}^{\min (k, n-1)} a_{j} b_{k-j}
$$

where $k=0, \ldots, m+n-2$.

## Correlation Operation

Closely related to convolution is the correlation operation. It computes the correlation of two sequences directly superposed or when one is shifted relative to the other. As with convolution, we can compute the correlation of two sequences efficiently as follows using the FFT:

- Compute the FFT of the two input sequences.
- Compute the pointwise product of the resulting transform of one sequence and the complex conjugate of the transform of the other sequence.
- Compute the inverse FFT of the product.

The routines in the Performance Library also allow correlation to be computed by the following definition:

$$
\operatorname{Corr}(x, y)_{j} \equiv \sum_{k=0}^{N-1} x_{j+k} y_{k}, \quad j=0, \ldots, N-1
$$

There are various ways to interpret the sampled input data of the convolution and correlation operations. The argument list of the convolution and correlation routines contain parameters to handle the following cases:

- The signal and/or response function can start at different sampling times.
- You might want only part of the signal to contribute to the output.
- The signal and/or response function can begin with one or more zeros that are not explicitly stored.


## Oracle Developer Studio Performance Library Convolution and Correlation Routines

Oracle Developer Studio Performance Library contains the convolution routines shown in Table 19, "Convolution and Correlation Routines," on page 115.

TABLE 19 Convolution and Correlation Routines

| Routine | Arguments | Function |
| :---: | :---: | :---: |
| SCNVCOR, DCNVCOR, CCNVCOR, ZCNVCOR | CNVCOR,FOUR,NX,X, IFX, INCX,NY, NPRE,M,Y,IFY, INC1Y,INC2Y,NZ,K, Z, IFZ,INC1Z, INC2Z,WORK, LWORK | Convolution or correlation of a filter with one or more vectors |
| SCNVCOR2, DCNVCOR2, CCNVCOR2, ZCNVCOR2 | CNVCOR,METHOD,TRANSX, SCRATCHX, TRANSY, SCRATCHY,MX,NX,X,LDX, MY,NY,MPRE,NPRE,Y,LDY, MZ,NZ,Z, LDZ,WORKIN, LWORK | Two-dimensional convolution or correlation of two matrices |
| SWIENER, DWIENER | N_POINTS,ACOR,XCOR, FLTR,EROP, ISW, IERR | Wiener deconvolution of two signals |

The [S, D, C , Z] CNVCOR routines are used to compute the convolution or correlation of a filter with one or more input vectors. The [S, D, C, Z]CNVCOR2 routines are used to compute the twodimensional convolution or correlation of two matrices.

## Arguments for Convolution and Correlation Routines

The one-dimensional convolution and correlation routines use the arguments shown in Table 20, "Arguments for One-Dimensional Convolution and Correlation Routines SCNVCOR, DCNVCOR, CCNVCOR, and ZCNVCOR," on page 115.

TABLE 20 Arguments for One-Dimensional Convolution and Correlation Routines SCNVCOR, DCNVCOR, CCNVCOR, and ZCNVCOR

| Argument | Definition |
| :--- | :--- |
| CNVCOR | ' $V$ ' or ' $v$ ' specifies that convolution is computed. ' $R$ ' or ' $r$ ' specifies that correlation <br> is computed. |
| FOUR | ' $T^{\prime}$ or ' $t$ ' specifies that the Fourier transform method is used. ' $D$ ' or ' $d$ ' specifies <br> that the direct method is used, where the convolution or correlation is computed from <br> the definition of convolution and correlation. |


| Argument | Definition |
| :---: | :---: |
| NX | Length of filter vector, where $\mathrm{NX} \geq 0$. |
| X | Filter vector |
| IFX | Index of first element of X , where $\mathrm{NX} \geq \mathrm{IFX} \geq 1$ |
| INCX | Stride between elements of the vector in X , where INCX $>0$. |
| NY | Length of input vectors, where $\mathrm{NY} \geq 0$. |
| NPRE | Number of implicit zeros prefixed to the $Y$ vectors, where NPRE $\geq 0$. |
| M | Number of input vectors, where $\mathrm{M} \geq 0$. |
| Y | Input vectors. |
| IFY | Index of the first element of $Y$, where $N Y \geq$ IFY $\geq 1$ |
| INCIY | Stride between elements of the input vectors in Y , where INC1Y $>0$. |
| INC2Y | Stride between input vectors in Y , where INC2Y>0. |
| NZ | Length of the output vectors, where $N Z \geq 0$. |
| K | Number of $Z$ vectors, where $K \geq 0$. If $K<M$, only the first $K$ vectors will be processed. If $K>M$, all input vectors will be processed and the last $M-K$ output vectors will be set to zero on exit. |
| Z | Result vectors |
| IFZ | Index of the first element of $Z$, where $N Z \geq I F Z \geq 1$ |
| INC1Z | Stride between elements of the output vectors in Z , where INCYZ $>0$. |
| INC2Z | Stride between output vectors in Z, where INC2Z > 0 . |
| WORK | Work array |
| LWORK | Length of work array |

${ }^{\dagger}$ When the lengths of the two sequences to be convolved are similar, the FFT method is faster than the direct method. However, when one sequence is much larger than the other, such as when convolving a large time-series signal with a small filter, the direct method performs faster than the FFT-based method.

The two-dimensional convolution and correlation routines use the arguments shown in Table 21, "Arguments for Two-Dimensional Convolution and Correlation Routines SCNVCOR2, DCNVCOR2, CCNVCOR2, and ZCNVCOR2," on page 116.

TABLE 21 Arguments for Two-Dimensional Convolution and Correlation Routines SCNVCOR2, DCNVCOR2, CCNVCOR2, and ZCNVCOR2

| Argument | Definition |
| :---: | :---: |
| CNVCOR | ' $V$ ' or ' $v$ ' specifies that convolution is computed. ' $R$ ' or ' $r$ ' specifies that correlation is computed. |
| METHOD | ' $T$ ' or ' $t$ ' specifies that the Fourier transform method is used. ' $D$ ' or ' $d$ ' specifies that the direct method is used, where the convolution or correlation is computed from the definition of convolution and correlation. ${ }^{\dagger}$ |
| TRANSX | ' $N$ ' or ' $n$ ' specifies that $X$ is the filter matrix ' $T$ ' or ' $t$ ' specifies that the transpose of X is the filter matrix |


| Argument | Definition |
| :---: | :---: |
| SCRATCHX | ' $N$ ' or ' $n$ ' specifies that $X$ must be preserved ' $S$ ' or ' $s$ ' specifies that $X$ can be used for scratch space. The contents of $X$ are undefined after returning from a call where $X$ is used for scratch space. |
| TRANSY | ' $N$ ' or ' $n$ ' specifies that $Y$ is the input matrix ' $T$ ' or ' $t$ ' specifies that the transpose of Y is the input matrix |
| SCRATCHY | ' $N$ ' or ' $n$ ' specifies that $Y$ must be preserved ' $S$ ' or ' $s$ ' specifies that $Y$ can be used for scratch space. The contents of $X$ are undefined after returning from a call where $Y$ is used for scratch space. |
| MX | Number of rows in the filter matrix X , where MX $\geq 0$ |
| NX | Number of columns in the filter matrix X , where $\mathrm{NX} \geq 0$ |
| x | Filter matrix. $X$ is unchanged on exit when SCRATCHX is ' $N$ ' or ' $n$ ' and undefined on exit when SCRATCHX is ' S ' or ' s '. |
| LDX | Leading dimension of array containing the filter matrix X . |
| MY | Number of rows in the input matrix Y , where MY $\geq 0$. |
| NY | Number of columns in the input matrix Y , where $\mathrm{NY} \geq 0$ |
| MPRE | Number of implicit zeros prefixed to each row of the input matrix $Y$ vectors, where MPRE $\geq 0$. |
| NPRE | Number of implicit zeros prefixed to each column of the input matrix Y , where NPRE $\geq$ 0. |
| Y | Input matrix. Y is unchanged on exit when SCRATCHY is ' N ' or ' n ' and undefined on exit when SCRATCHY is ' S ' or ' s '. |
| LDY | Leading dimension of array containing the input matrix Y . |
| MZ | Number of output vectors, where MZ $\geq 0$. |
| NZ | Length of output vectors, where $N Z \geq 0$. |
| Z | Result vectors |
| LDZ | Leading dimension of the array containing the result matrix $Z$, where $L D Z \geq \operatorname{MAX}(1, M Z)$. |
| WORKIN | Work array |
| LWORK | Length of work array |

${ }^{\dagger}$ When the sizes of the two matrices to be convolved are similar, the FFT method is faster than the direct method. However, when one sequence is much larger than the other, such as when convolving a large data set with a small filter, the direct method performs faster than the FFT-based method.

## Work Array work for Convolution and Correlation Routines

The minimum dimensions for the WORK work arrays used with the one-dimensional and two-dimensional convolution and correlation routines are shown in Table 24, "Minimum Dimensions and Data Types for WORK Work Array Used With Convolution and Correlation

Routines," on page 119. The minimum dimensions for one-dimensional convolution and correlation routines depend upon the values of the arguments NPRE, NX, NY, and NZ.

The minimum dimensions for two-dimensional convolution and correlation routines depend upon the values of the arguments shown in the table Table 22, "Arguments Affecting Minimum Work Array Size for Two-Dimensional Routines: SCNVCOR2, DCNVCOR2, CCNVCOR2, and ZCNVCOR2," on page 118.

TABLE 22 Arguments Affecting Minimum Work Array Size for Two-Dimensional Routines: SCNVCOR2, DCNVCOR2, CCNVCOR2, and ZCNVCOR2

| Argument | Definition |
| :--- | :--- |
| MX | Number of rows in the filter matrix |
| MY | Number of rows in the input matrix |
| MZ | Number of output vectors |
| NX | Number of columns in the filter matrix |
| NY | Number of columns in the input matrix |
| NZ | Length of output vectors |
| MPRE | Number of implicit zeros prefixed to each row of the input matrix |
| NPRE | Number of implicit zeros prefixed to each column of the input matrix |
| MPOST | MAX(0,MZ-MYC) |
| NPOST | MAX(0,NZ-NYC) |
| MYC | MPRE + MPOST + MYC_INIT, where MYC_INIT depends upon filter and input matrices, as |
|  | shown in Table 23, "MYC_INIT and NYC_INIT Dependencies," on page 118 |
| NYC | NPRE + NPOST + NYC_INIT, where NYC_INIT depends upon filter and input matrices, as |
|  | shown in Table 23, "MYC_INIT and NYC_INIT Dependencies," on page 118 |

MYC_INIT and NYC_INIT depend upon the following, where $X$ is the filter matrix and $Y$ is the input matrix.

TABLE 23 MYC_INIT and NYC_INIT Dependencies

| Routine | Y |  |  | Transpose(Y) |
| :--- | :--- | :--- | :--- | :---: |
|  | X | Transpose(X) | X | Transpose(X) |
| MYC_INIT | MAX(MX,MY) | MAX(NX,MY) | MAX(MX,NY) | MAX(NX,NY) |
| NYC_INIT | MAX(NX,NY) | MAX(MX,NY) | MAX(NX,MY) | MAX(MX,MY) |

The values assigned to the minimum work array size is shown in Table 24, "Minimum Dimensions and Data Types for WORK Work Array Used With Convolution and Correlation Routines," on page 119.

TABLE 24 Minimum Dimensions and Data Types for WORK Work Array Used With Convolution and Correlation Routines

| Routine | Minimum Work Array Size (WORK) | Type |
| :--- | :--- | :--- |
| SCNVCOR, DCNVCOR | $4^{*}($ MAX $(N X, N P R E+N Y)+\operatorname{MAX}(0, N Z-N Y))$ | REAL, REAL*8 |
| CCNVCOR, ZCNVCOR | $\left.2^{*}(M A X(N X, N P R E+N Y)+\operatorname{MAX}(0, N Z-N Y))\right)$ | COMPLEX, COMPLEX*16 |
| SCNVCOR2 $^{\dagger}$, DCNVCOR2 $^{\dagger}$ | $M Y+N Y+30$ | COMPLEX, COMPLEX*16 |
| CCNVCOR2 $^{\dagger}$, ZCNVCOR2 $^{\dagger}$ | If MY $=N Y: M Y C+8$ If MY $\geq$ NY: MYC + NYC +16 | COMPLEX, COMPLEX*16 |

1

## Sample Program: Convolution

The following example uses CCNVCOR to perform FFT convolution of two complex vectors.

EXAMPLE 19 One-Dimensional Convolution Using Fourier Transform Method and COMPLEX Data
my_system\% cat con_ex20.f
PROGRAM TEST
C

| INTEGER | LWORK |
| :--- | :---: |
| INTEGER | N |
| PARAMETER | $(\mathrm{N}=3)$ |

PARAMETER (LWORK $=4 * \mathrm{~N}+15$ )
COMPLEX P1(N), P2(N), P3(2*N-1), WORK(LWORK)
DATA P1 / 1, 2, 3 /, P2 / 4, 5, 6 /
C
EXTERNAL CCNVCOR
C
PRINT *, 'P1:'
PRINT 1000, P1
PRINT *, 'P2:'
PRINT 1000, P2
CALL CCNVCOR ('V', 'T', N, P1, 1, 1, N, 0, 1, P2, 1, 1, 1,
\$ 2 * N $-1,1$, P3, 1, 1, 1, WORK, LWORK)
C
PRINT *, 'P3:'
PRINT 1000, P3
C
1000 FORMAT (1X, 100(F4.1,' +',F4.1,'i '))
C

[^0]END

```
my_system% f95 -dalign con_ex20.f -xlibrary=sunperf
my_system% a.out
    P1:
        1.0+0.0i 2.0+0.0i 3.0+0.0i
    P2:
        4.0 + 0.0i 5.0 + 0.0i 6.0 + 0.0i
    P3:
        4.0+0.0i 13.0+0.0i 28.0 + 0.0i 27.0 + 0.0i 18.0 + 0.0i
```

If any vector overlaps a writable vector, either because of argument aliasing or ill-chosen values of the various INC arguments, the results are undefined and can vary from one run to the next.

The most common form of the computation, and the case that executes fastest, is applying a filter vector $X$ to a series of vectors stored in the columns of $Y$ with the result placed into the columns of $Z$. In that case, $I N C X=1, I N C 1 Y=1, I N C 2 Y \geq N Y, I N C 1 Z=1, I N C 2 Z \geq N Z$. Another common form is applying a filter vector $X$ to a series of vectors stored in the rows of $Y$ and store the result in the row of $Z$, in which case $\operatorname{INCX}=1$, INC1Y $\geq N Y$, INC2Y $=1$, INC1Z $\geq N Z$, and INC2Z $=1$.

Convolution can be used to compute the products of polynomials. The following example Example 20, "One-Dimensional Convolution Using Fourier Transform Method and REAL Data," on page 120 uses SCNVCOR to compute the product of $1+2 x+3 x^{2}$ and $4+5 x+6 x^{2}$.

EXAMPLE 20 One-Dimensional Convolution Using Fourier Transform Method and REAL Data

```
my_system% cat con_ex21.f
    PROGRAM TEST
    INTEGER LWORK, NX, NY, NZ
    PARAMETER (NX = 3)
    PARAMETER (NY = NX)
    PARAMETER (NZ = 2*NY-1)
    PARAMETER (LWORK = 4*NZ+32)
    REAL X(NX), Y(NY), Z(NZ), WORK(LWORK)
C
    DATA X / 1, 2, 3 /, Y / 4, 5, 6 /, WORK / LWORK*0 /
C
    PRINT 1000, 'X'
    PRINT 1010, X
    PRINT 1000, 'Y'
    PRINT 1010, Y
    CALL SCNVCOR ('V', 'T', NX, X, 1, 1,
    $NY, 0, 1, Y, 1, 1, 1, NZ, 1, Z, 1, 1, 1, WORK, LWORK)
        PRINT 1020, 'Z'
        PRINT 1010, Z
1000 FORMAT (1X, 'Input vector ', A1)
1010 FORMAT (1X, 300F5.0)
```

```
1020 FORMAT (1X, 'Output vector ', Al)
    END
my_system% f95 -dalign con_ex21.f -library=sunperf
my_system% a.out
    Input vector X
        1. 2. 3.
Input vector Y
    4. 5. 6.
Output vector Z
    4. 13. 28. 27. 18.
```

Making the output vector longer than the input vectors, as in the example above, implicitly adds zeros to the end of the input. No zeros are actually required in any of the vectors, and none are used in the example, but the padding provided by the implied zeros has the effect of an end-off shift rather than an end-around shift of the input vectors.

The following example Example 21, "Convolution Used to Compute the Product of a Vector and Circulant Matrix," on page 121 computes the product between the vector [ 1, 2, 3 ] and the circulant matrix defined by the initial column vector [ $4,5,6$ ].

EXAMPLE 21 Convolution Used to Compute the Product of a Vector and Circulant Matrix

```
my_system% cat con_ex22.f
    PROGRAM TEST
C
    INTEGER LWORK, NX, NY, NZ
    PARAMETER (NX = 3)
    PARAMETER (NY = NX)
    PARAMETER (NZ = NY)
    PARAMETER (LWORK = 4*NZ+32)
    REAL X(NX),Y(NY), Z(NZ), WORK(LWORK)
C
    DATA X / 1, 2, 3 /, Y / 4, 5, 6 /, WORK / LWORK*0 /
C
    PRINT 1000, 'X'
    PRINT 1010, X
    PRINT 1000, 'Y'
    PRINT 1010, Y
    CALL SCNVCOR ('V', 'T', NX, X, 1, 1,
    $NY, 0, 1, Y, 1, 1, 1, NZ, 1, Z, 1, 1, 1,
    $WORK, LWORK)
        PRINT 1020, 'Z'
        PRINT 1010, Z
C
    1000 FORMAT (1X, 'Input vector ', Al)
    1010 FORMAT (1X, 300F5.0)
    1020 FORMAT (1X, 'Output vector ', Al)
```


## END

```
my_system% f95 -dalign con_ex22.f -library=sunperf
my_system% a.out
    Input vector X
        1. 2. 3.
    Input vector Y
            4. 5. 6.
    Output vector Z
        31. 31. 28.
```

The difference between the following example and the previous example is that the length of the output vector is the same as the length of the input vectors, so there are no implied zeros on the end of the input vectors. With no implied zeros to shift into, the effect of an end-off shift from the previous example does not occur and the end-around shift results in a circulant matrix product.

## EXAMPLE 22 Two-Dimensional Convolution Using Direct Method

```
my_system% cat con_ex23.f
    PROGRAM TEST
C
    INTEGER M, N
    PARAMETER (M = 2)
    PARAMETER (N = 3)
C
    INTEGER I, J
    COMPLEX P1(M,N), P2(M,N), P3(M,N)
    DATA P1 / 1, -2, 3, -4, 5, -6 /, P2 / -1, 2, -3, 4, -5, 6 /
    EXTERNAL CCNVCOR2
C
    PRINT *, 'P1:'
    PRINT 1000, ((Pl(I,J), J = 1, N), I = 1, M)
    PRINT *, 'P2:'
    PRINT 1000, ((P2(I,J), J = 1, N), I = 1, M)
C
    CALL CCNVCOR2 ('V', 'Direct', 'No Transpose X', 'No Overwrite X',
    $ 'No Transpose Y', 'No Overwrite Y', M, N, P1, M,
    $ M, N, 0, 0, P2, M, M, N, P3, M, 0, 0)
C
    PRINT *, 'P3:'
    PRINT 1000, ((P3(I,J), J = 1, N), I = 1, M)
C
    1000 FORMAT (3(F5.1,' +',F5.1,'i '))
C
    END
my_system% f95 -dalign con_ex23.f -library=sunperf
my_system% a.out
```

| P1: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1.0+$ | $0.0 i$ | $3.0+$ | 0.0i | $5.0+$ | $0.0 i$ |
| $-2.0+$ | $0.0 i$ | $-4.0+$ | 0.0i | $-6.0+$ | 0.01 |
| P2: |  |  |  |  |  |
| $-1.0+$ | $0.0 i$ | $-3.0+$ | 0.0i | $-5.0+$ | $0.0 i$ |
| $2.0+$ | $0.0 i$ | $4.0+$ | 0.0i | $6.0+$ | 0.01 |
| P3: |  |  |  |  |  |
| $-83.0+$ | $0.0 i$ | $-83.0+$ | $0.0 i$ | $-59.0+$ | 0.01 |
| 80.0 + | $0.0 i$ | 80.0 + | 0.0i | 56.0 + | 0.01 |

## References

For additional information on the DFT or FFT, see the following sources.
Briggs, William L., and Henson, Van Emden. The DFT: An Owner's Manual for the Discrete Fourier Transform. Philadelphia, PA: SIAM, 1995.

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Press, William H., Teukolsky, Saul A., Vetterling, William T., and Flannery, Brian P. Numerical Recipes in C: The Art of Scientific Computing. 2 ed. Cambridge, United Kingdom: Cambridge University Press, 1992.

Ramirez, Robert W. The FFT: Fundamentals and Concepts. Englewood Cliffs, NJ: PrenticeHall, Inc., 1985.

Swartzrauber, Paul N. Vectorizing the FFTs. In Rodrigue, Garry ed. Parallel Computations. New York: Academic Press, Inc., 1982.

Strang, Gilbert. Linear Algebra and Its Applications. 3 ed. Orlando, FL: Harcourt Brace \& Company, 1988.

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Walker, James S. Fast Fourier Transforms. Boca Raton, FL: CRC Press, 1991.

## Oracle Developer Studio Performance Library Routines

This appendix lists the Oracle Developer Studio Performance Library routines by library, routine name, and function.

For a description of the function and a listing of the Fortran and C interfaces, refer to the section 3P man pages for the individual routines. For example, to display the man page for the SBDSQR routine, type man -s 3P sbdsqr. The man page routine names use lowercase letters.

For many routines, separate routines exist that operate on different data types. Rather than list each routine separately, a lowercase $x$ is used in a routine name to denote single, double, complex, and double complex data types. For example, the routine $x$ BDSQR is available as four routines that operate with the following data types:

- SBDSQR - Single data type
- DBDSQR - Double data type
- CBDSQR - Complex data type
- ZBDSQR - Double complex data type

If a routine name is not available for $\mathrm{S}, \mathrm{D}, \mathrm{C}$, and Z , the x prefix will not be used and each routine name will be listed. Also available (but not listed) in 64-bit enable operating environments are the corresponding routines in 64-bit. Their names are denoted by the _64 suffix. For example, the 64-bit versions of $x$ BDSQR are the following:

- SBDSQR_64
- DBDSQR_64
- CBDSQR_64
- ZBDSQR_64


## LAPACK Routines

The following set of tables lists the Oracle Developer Studio Performance Library LAPACK routines. ( P ) denotes routines that are parallelized.

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TABLE 25 Bidiagonal Matrix Routines

| Routine | Function |
| :--- | :--- |
| SBDSDC (P) or DBDSDC (P) | Computes the singular value decomposition (SVD) of a bidiagonal matrix, using a <br> divide and conquer method. <br> SBDSVDX or DBDSVDX <br> Computes the singular value decomposition (SVD) of a real N-by-N (upper or lower) <br> bidiagonal matrix (driver). <br> Computes SVD of a real upper or lower bidiagonal matrix, using the implicit zero-shift <br> QR algorithm. |

TABLE 26 Common or Calculating Routines

| Routine | Function |
| :---: | :---: |
| CHLA_TRANSTYPE | Translates from a BLAST-specified integer constant to the character string specifying a transposition operation. |
| CLA_HERPVGRW (P) or | Computes the reciprocal pivot growth factor norm(A)/norm(U) for a complex |
| ZLA_HERPVGRW (P) | Hermitian matrix. |
| ILADIAG | Translates from a character string specifying, if a matrix has the unit diagonal or not, to the relevant BLAST-specified integer constant. |
| ILAPREC | Translates from a character string specifying an intermediate precision to the relevant BLAST-specified integer constant. |
| ILATRANS | Translates from a character string specifying a transposition operation to the relevant BLAST-specified integer constant. |
| ILAENV | Is called from the LAPACK routines to choose problem-dependent parameters for the local environment. |
| ILAUPLO | Translates from a character string specifying an upper or lower triangular matrix to the relevant BLAST-specified integer constant. |
| ILAVER | Returns the LAPACK version. |
| $x$ LA_GBRPVGRW | Computes the reciprocal pivot growth factor norm(A)/norm(U) for a real or complex general banded matrix. |
| $x$ LA_GERPVGRW (P) | Computes the reciprocal pivot growth factor norm(A)/norm(U) for a general indefinite matrix. |
| $x$ LA_PORPVGRW (P) | Computes the reciprocal pivot growth factor norm(A)/norm(U) for a real symmetric or Hermitian positive definite matrix. |
| $x$ LA_SYRPVGRW (P) | Computes the reciprocal pivot growth factor norm(A)/norm(U) for a real or complex symmetric indefinite matrix. |


| Routine | Function |
| :---: | :---: |
| SLAMRG (P) or DLAMRG (P) | Creates a permutation list to merge the entries of two independently sorted sets into a single set sorted in ascending order. |
| CLANHF (P) or ZLANHF (P) | Returns a value of the one-norm, Frobenius norm, infinity norm, or the element of largest absolute value of a Hermitian matrix in the RFP format. |
| SLANSF (P) or DLANSF (P) | Returns a value of the one-norm, Frobenius norm, infinity norm, or the element of largest absolute value of a real symmetric matrix in the RFP format. |
| $x$ LARSCL2 (P) | Performs a reciprocal diagonal scaling on a vector. |
| $x$ LASCL2 (P) | Performs a diagonal scaling on a vector. |
| SLASQ1 or DLASQ1 | Computes the singular values of a real square bidiagonal matrix. Used by SBDSQR or DBDSQR. |
| SLASQ2 or DLASQ2 | Computes all the eigenvalues of a real symmetric positive definite tridiagonal matrix (high relative accuracy). Used by SBDSQR and SSTEGR or DBDSQR and DSTEGR. |
| SLASQ3 or DLASQ3 | Checks for deflation, computes a shift and calls the DQDS algorithm. Used by SBDSQR or DBDSQR. |
| SLASQ4 or DLASQ4 | Computes an approximation to the smallest eigenvalue using values from the previous transform. Used by SBDSQR or DBDSQR. |
| SLASQ5 or DLASQ5 | Computes one DQDS transform in the ping-pong form. Used by SBDSQR and SSTEGR or DBDSQR and DSTEGR. |
| SLASQ6 or DLASQ6 | Computes one DQD transform (shift equal to zero) in ping-pong form, with protection against underflow and overflow. Used by SBDSQR and SSTEGR or DBDSQR and DSTEGR. |
| SLASRT or DLASRT | Sorts numbers in a vector in increasing or decreasing order. |
| $x$ LATRZ (P) | Factors a real or complex upper trapezoidal matrix by means of orthogonal transformations. |
| CROT, ZROT | Apply Givens plane rotation Note that SROT/DROT are included in level 1 BLAS. |

TABLE 27 Cosine-Sine (CS) Decomposition Routines

| Routine | Function |
| :--- | :--- |
| $x$ BBCSD (P) | Computes the CS decomposition of an unitary or orthogonal matrix in a bidiagonal- <br> block form. |
| SORCSD (P) or DORCSD (P) <br> SORCSD2BY1 or DORCSD2BY1 <br> $(P)$ | Computes the CS decomposition of a real partitioned orthogonal matrix. <br> Computes the CS decomposition of an M-by-Q matrix X with orthonormal columns <br> that has been partitioned into a 2-by-1 block structure. |
| CUNCSD (P) or ZUNCSD (P) | Computes the CS decomposition of an M-by-M partitioned unitary matrix. |

TABLE 28 Diagonal Matrix Routines

| Routine | Function |
| :--- | :--- |
| SDISNA (P) or DDISNA (P) | Computes the reciprocal of the condition numbers for eigenvectors of a real symmetric <br> or complex Hermitian matrix. |

TABLE 29 General Band Matrix Routines

| Routine | Function |
| :---: | :---: |
| CGBBRD or ZGBBRD | Reduces a complex general band matrix to an upper bidiagonal form by the orthogonal transformation. |
| SGBBRD (P) or DGBBRD (P) | Reduces a real general band matrix to an upper bidiagonal form by the orthogonal transformation. |
| $x$ GBCON | Estimates the reciprocal of the condition number of a general band matrix using LU factorization. |
| $x$ GBEQU (P) | Computes row and column scalings to equilibrate a general band matrix and reduce its condition number. |
| $x$ GBEQUB (P) | Computes row and column scalings intended to equilibrate a general band matrix and reduce its condition number. Differs from CGEEQU by restricting the scaling factors to a power of the radix. |
| $x$ GBRFS (P) | Improves the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution. |
| $x$ GBRFSX (P) | Improves the computed solution to a banded system of linear equations and provides error bounds and backward error estimates. In addition to normwise error bound, the code provides maximum componentwise error bound if possible. |
| $x$ GBSV | Solves a general banded system of linear equations (simple driver). |
| $x \mathrm{GBSVX}$ (P) | Solves a general banded system of linear equations (expert driver). |
| $x$ GBSVXX (P) | Solves a general banded system of linear equations (expert driver, extra precision). If requested, both normwise and maximum componentwise error bounds are returned. |
| $x$ GBTF2 (P) | Computes the LU factorization of a real or complex general band matrix using partial pivoting with row interchanges (unblocked algorithm). |
| $\chi$ GBTRF (P) | Computes the LU factorization of a general band matrix using partial pivoting with row interchanges. |
| $x$ GBTRS | Solves a general banded system of linear equations, using the factorization computed by $x$ GBTRF. |
| $x$ LA_GBAMV | Performs a matrix-vector operation to calculate error bounds for a real or complex band matrix. |
| xLA_GBRFSX_EXTENDED | Improves the computed solution to a system of linear equations for a real or complex general banded matrix by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. |

TABLE 30 General Matrix (Unsymmetric or Rectangular) Routines

| Routine | Function |
| :--- | :--- |
| $x G E J S V(P)$ | Computes the singular value decomposition (SVD) of a real or complex general matrix. |
| DSGESV | Computes the solution to a real system of linear equations with a general matrices <br> (mixed precision with iterative refinement). |


| Routine | Function |
| :---: | :---: |
| xGESVJ (P) | Computes the singular value decomposition (SVD) of a real or complex general matrix. Implements a preconditioned Jacobi SVD algorithm. Uses $x G E Q P 3, x G E Q R, x G E L Q F$ or $x$ KEQP3 as a preprocessor. |
| ZCGESV | Computes the solution to a complex system of linear equations with a general matrices (mixed precision with iterative refinement). |
| zCPOSV | Computes the solution to a complex system of linear equations with a positive definite matrix (mixed precision with iterative refinement). |
| xGEBAK | Forms the right or left eigenvectors of a general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by $x$ GEBAL. |
| $x$ GEBAL (P) | Balances a real or complex general matrix. |
| $x$ GEBD2 | Reduces a general matrix to bidiagonal form (unblocked algorithm). |
| xGEBRD | Reduces a general matrix to upper or lower bidiagonal form by an unitary or orthogonal transformation (blocked algorithm). |
| xGECON | Estimates the reciprocal of the condition number of a general matrix, using the factorization computed by $x$ GETRF. |
| $x$ GEEQU (P) | Computes row and column scalings intended to equilibrate a general rectangular matrix and reduce its condition number. |
| $x$ GEEQUB (P) | Computes row and column scalings intended to equilibrate a general rectangular matrix and reduce its condition number. Differs from $x$ GETRF by restricting the scaling factors to a power of the radix. |
| $\chi$ GEES | Computes the eigenvalues and Schur factorization of a general matrix (simple driver). |
| xGEESX | Computes the eigenvalues and Schur factorization of a general matrix (expert driver). |
| $x G E E V$ (P) | Computes the eigenvalues and left and right eigenvectors of a general matrix (simple driver). |
| $x \mathrm{GEEVX}$ (P) | Computes the eigenvalues and left and right eigenvectors of a general matrix (expert driver). |
| xGEGS | Deprecated routine replaced by $x$ GGES. |
| $x$ GEGV (P) | Deprecated routine replaced by $x$ GGEV. |
| xGEHD2 | Reduces a general square matrix to an upper Hessenberg form by the unitary or orthogonal similarity transformation (unblocked algorithm). |
| $x$ GEHRD (P) | Reduces a general matrix to upper Hessenberg form by an orthogonal similarity transformation. |
| xGELQ2 | Computes the LQ factorization of a real or complex general rectangular matrix (unblocked algorithm). |
| XGELQF | Computes the LQ factorization of a general rectangular matrix. |
| $x$ GELS (P) | Computes the least squares solution to an over-determined system of linear equations using a QR or LQ factorization of A. |
| $x$ GELSD | Computes the least squares solution to an over-determined system of linear equations using a divide and conquer method and a QR or LQ factorization of A . |
| xGELSS | Computes the minimum-norm solution to a linear least squares problem by using the SVD of a general rectangular matrix (simple driver). |


| Routine | Function |
| :---: | :---: |
| xGELSX (P) | Deprecated routine replaced by $x$ SELSY. |
| $x G E L S Y$ (P) | Computes the minimum-norm solution to a linear least squares problem using a complete orthogonal factorization. |
| $\chi$ GEMQRT | Overwrites a general matrix with the result of its transformation by an orthogonal matrix, defined as the product of elementary reflectors generated using the compact WY representation as returned by $x G E Q R T$. |
| xGEQL2 | Computes the QL factorization of a real or complex general rectangular matrix (unblocked algorithm). |
| xGEQLF | Computes the QL factorization of a real or complex general rectangular matrix. |
| xGEQP3 | Computes the QR factorization of general rectangular matrix using Level 3 BLAS. |
| $x$ GEQPF | Deprecated routine replaced by $\chi$ GEQP3. |
| xGEQR2 | Computes the QR factorization of a real or complex general rectangular matrix (unblocked algorithm). |
| xGEQR2P | Computes the QR factorization of a real or complex general rectangular matrix with non-negative diagonal elements (unblocked algorithm). |
| XGEQRFP | Computes the QR factorization of a real or complex general rectangular matrix. |
| $x$ GEQRT | Computes a blocked QR factorization of a general real or complex matrix using the compact WY representation of Q . |
| xGEQRT2 | Computes a QR factorization of a general real or complex matrix using the compact WY representation of Q . |
| xGEQRT3 (P) | Recursively computes a QR factorization of a general real or complex matrix using the compact WY representation of Q. |
| $x$ GERFS (P) | Refines the solution to a system of linear equations. |
| $x$ gerfs ( P ) | Improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution (extra precision). |
| xGERQ2 | Computes the RQ factorization of a real or complex general rectangular matrix using an unblocked algorithm. |
| XGERQF | Computes the RQ factorization of a real or complex general rectangular matrix. |
| $\chi$ GESDD | Computes the singular value decomposition (SVD) of a real or complex general rectangular matrix using a divide and conquer method (driver). |
| xGESV | Solves a general system of linear equations (simple driver). |
| $\chi$ GESVD | Computes the singular value decomposition (SVD) for a real or complex general matrix (driver). |
| $x$ gESVDX | Computes the singular value decomposition (SVD) for a real or complex general matrix, allows the computation of a subset of singular values and vectors (driver). |
| xGESVJ | Computes the singular value decomposition (SVD) of a real or complex general rectangular matrix. |
| xGESVX (P) | Solves a general system of linear equations (expert driver). |
| $x G E S V X X ~(P)$ | Computes the solution to a system of linear equations for general matrices (extra precision). |


| Routine | Function |
| :---: | :---: |
| xGETF2 | Computes the LU factorization of a real or complex general matrix using partial pivoting with row interchanges (unblocked algorithm). |
| $x$ gETRF (P) | Computes the LU factorization of a real or complex general rectangular matrix using partial pivoting with row interchanges. |
| xGETRF2 (P) | Computes the LU factorization of a real or complex general rectangular matrix using partial pivoting with row interchanges (recursive algorithm). |
| $\chi$ GETRI | Computes the inverse of a general matrix using the factorization computed by xGETRF. |
| $x$ GETRS | Solves a general system of linear equations using the factorization computed by $x$ GETRF. |
| xGSVJ0 (P) | Preprocessor for $\chi$ GESVJ. Applies Jacobi rotations targeting only particular pivots. |
| $x$ ¢SVJ1 (P) | Preprocessor for $x$ GESVJ. Applies Jacobi rotations in the same way as $x$ GESVJ does, but it does not check convergence (stopping criterion). |
| $x$ LA_GEAMV (P) | Performs a matrix-vector operation to calculate error bounds for a real or complex general matrix. |
| CLA_GERCOND_C (P) or ZLA_GERCOND_C (P) | Computes the infinity norm condition number of op(A)*inv(diag(c)) for a complex general matrix. C is a REAL vector. |
| $\begin{aligned} & \text { CLA_GERCOND_X (P) or } \\ & \text { ZLA_GERCOND_X (P) } \end{aligned}$ | Computes the infinity norm condition number of op(A)*inv(diag(x)) for a complex general matrix. X is a COMPLEX vector. |
| SLA_GERCOND(P) or DLA_GERCOND (P) | Estimates the Skeel condition number for a real general matrix. |
| xLA_GERFSX_EXTENDED ( P ) | Improves the computed solution to a system of linear equations for a real or complex general matrix by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. |
| $x$ LA_GERFSX_GBRPVGRW | Computes the reciprocal pivot growth factor norm(A)/norm(U) for a real or complex general matrix. |
| $x$ LALS0 (P) | Applies back multiplying factors in solving the least squares problem using the divide and conquer SVD approach. Used by $x$ LALSA. |
| CLALSA (P) or ZLALSA (P) | Computes the SVD of a complex matrix in compact form. Used by SGELSD. |
| SLALSA or DLALSA | Computes the SVD of a real matrix in compact form. Used by SGELSD or DGELSD. |
| $x$ LALSD (P) | Solves the least squares problem using the SVD. Used by xGELSD. |

TABLE 31 General Matrix-Generalized Problem (Pair of General Matrices) Routines

| Routine | Function |
| :--- | :--- |
| $x$ GGBAK | Forms the right or left eigenvectors of a generalized eigenvalue problem based on the <br> output by $x$ <br> $x G B B A L$. |
| $x$ GGBAL (P) | Balances a pair of general matrices for the generalized eigenvalue problem. <br> Computes the generalized eigenvalues, Schur form, and, optionally, left and/or right <br> Schur vectors for two nonsymmetric matrices (simple driver). |


| Routine | Function |
| :---: | :---: |
| xGGES3 | Computes the generalized eigenvalues, Schur form, and, optionally, left and/or right Schur vectors for two nonsymmetric matrices using a blocked algorithm (simple driver). |
| XGGESX | Computes the generalized eigenvalues, Schur form, and, optionally, left and/or right Schur vectors (expert driver). |
| $x$ GGEV (P) | Computes the generalized eigenvalues and the left and/or right generalized eigenvectors for two nonsymmetric matrices (simple driver). |
| $x$ GGEV3 | Computes the generalized eigenvalues and the left and/or right generalized eigenvectors for two nonsymmetric matrices using a blocked algorithm (expert driver). |
| $x$ GGEVX (P) | Computes the generalized eigenvalues and the left and/or right generalized eigenvectors for two nonsymmetric matrices (expert driver). |
| $x$ GGGLM (P) | Solves the general Gauss-Markov linear model (GLM) problem. |
| $x$ GGHD3 (P) | Reduces two matrices to the generalized upper Hessenberg form using orthogonal transformations. This is a blocked variant of xGGHRD used to enhance performance. |
| $x$ GGHRD (P) | Reduces two matrices to the generalized upper Hessenberg form using orthogonal transformations. |
| xGGLSE | Solves the LSE (Constrained Linear Least Squares Problem) using the GRQ (Generalized RQ) factorization. |
| $x$ GGQRF | Computes the generalized QR factorization of two matrices. |
| XGGRQF | Computes the generalized RQ factorization of two matrices. |
| $x$ GGSVD | Computes the generalized singular value decomposition (driver). |
| $x$ GGSVD3 | Computes the generalized singular value decomposition (driver). |
| $x G G S V P$ (P) | Computes an orthogonal or unitary matrix as a preprocessing step for calculating the generalized singular value decomposition using xGGSVD. |
| $x$ GGSVP3 (P) | Computes an orthogonal or unitary matrix as a preprocessing step for calculating the generalized singular value decomposition using $x$ GGSVD3. |

TABLE 32 General Tridiagonal Matrix Routines

| Routine | Function |
| :--- | :--- |
| $x$ GTCON | Estimates the reciprocal of the condition number of a tridiagonal matrix, using the LU <br> factorization as computed by $x$ GTTRF. |
| $x$ RTRFS (P) | Refines the solution to a general tridiagonal system of linear equations. <br> Solves a general tridiagonal system of linear equations (simple driver). |
| $x$ GTSVX |  |
| $x$ GTTRF (P) | Solves a general tridiagonal system of linear equations (expert driver). <br> Computes an LU factorization of a general tridiagonal matrix using partial pivoting and <br> row exchanges. <br> Solves general tridiagonal system of linear equations using the factorization computed <br> by $x$ GTTRF. <br> Solves a system of linear equations with a tridiagonal matrix using the LU factorization <br> computed by $x$ GTTRF. |
| $x$ GTTS2 (P) |  |

TABLE 33 Hermitian Band Matrix Routines

| Routine | Function |
| :--- | :--- |
| CHBEV or ZHBEV | Computes all the eigenvalues and eigenvectors of a Hermitian band matrix. <br> Replacement with newer version CHBEVD or ZHBEVD suggested. <br> Computes all the eigenvalues and eigenvectors of a Hermitian band matrix and uses a <br> divide and conquer method to calculate eigenvectors (driver). <br> Computes selected eigenvalues and eigenvectors of a Hermitian band matrix. |
| CHBEVX (P) or ZHBEVX (P) | Reduces Hermitian-definite banded generalized eigenproblem to a standard form. <br> CHBGST (P) or ZHBGST (P) <br> CHBGV or ZHBGV <br> CHBGVD or ZHBGVD <br> CHBEVGVX (P) or ZHBGVX (P) <br> banded eigenproblem. Replacement with newer version CHBGVD or ZHBGVD suggested. <br> Computes all the eigenvalues and eigenvectors of a generalized Hermitian-definite <br> banded eigenproblem and uses a divide and conquer method to calculate eigenvectors <br> (driver). <br> Computes selected eigenvalues and eigenvectors of a generalized Hermitian-definite <br> banded eigenproblem. <br> Reduces a Hermitian band matrix to a real symmetric tridiagonal form by using a <br> unitary similarity transformation. |

## TABLE 34 Hermitian Matrix Routines

\(\left.$$
\begin{array}{l|l}\hline \text { Routine } & \text { Function } \\
\hline \text { CHECON or ZHECON } & \begin{array}{l}\text { Estimates the reciprocal of the condition number of a Hermitian matrix using the } \\
\text { factorization computed by CHETRF or ZHETRF. } \\
\text { EHECON_ROOK or } \\
\text { ZHECON_ROOK }\end{array} \\
\hline \begin{array}{l}\text { EHEEQUB (P) or ZHEEQUB } \\
\text { factorization computed by CHETRF_ROOK or ZHETRF_ROOK. }\end{array} \\
\text { CHEEV or ZHEEV } & \begin{array}{l}\text { Computes row and column scalings intended to equilibrate a Hermitian matrix and } \\
\text { reduce its condition number with respect to the two-norm. }\end{array} \\
\text { CHEEVD or ZHEEVD } & \begin{array}{l}\text { Computes all the eigenvalues and eigenvectors of a Hermitian matrix (simple driver). } \\
\text { Replacement with newer version CHEEVR or ZHEEVR suggested. } \\
\text { Computes all the eigenvalues and eigenvectors of a Hermitian matrix and uses a }\end{array}
$$ <br>
divide and conquer method to calculate eigenvectors (driver). Replacement with newer <br>
version CHEEVR or ZHEEVR suggested. <br>

Computes selected eigenvalues and the eigenvectors of a complex Hermitian matrix.\end{array}\right]\)| Computes selected eigenvalues and eigenvectors of a Hermitian matrix (expert driver). |
| :--- |
| CHEEVX (P) or ZHEEVX (P) |
| CHEGST or ZHEGST |
| CHEGVD or ZHEGVD |


| Routine | Function |
| :---: | :---: |
| CHEGVX or ZHEGVX | Computes selected eigenvalues and eigenvectors of a complex generalized Hermitiandefinite eigenproblem. |
| CHERFS (P) or ZHERFS (P) | Improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite. |
| CHERFSX (P) or ZHERFSX (P) | Improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite (extra precision). |
| CHESV or ZHESV | Solves a complex Hermitian-indefinite system of linear equations (simple driver). CHETRF is called to compute the factorization of a complex Hermitian matrix |
| CHESV_ROOK or ZHESV_ROOK | Solves a complex Hermitian-indefinite system of linear equations (simple driver). CHETRF_ROOK is called to compute the factorization of a complex Hermitian matrix. |
| CHESVX or ZHESVX | Solves a complex Hermitian-indefinite system of linear equations (expert driver). |
| CHESVXX (P) or ZHESVXX (P) | Computes the solution to a complex system of linear equations with a square symmetric matrix using the diagonal pivoting factorization (extra precision). |
| CHETD2 or ZHETD2 | Reduces a complex Hermitian matrix to a real symmetric tridiagonal form by an unitary similarity transformation (an unblocked algorithm). |
| CHETF2 (P) or ZHETF2 (P) | Computes the factorization of a complex Hermitian matrix using the diagonal pivoting method (unblocked algorithm). |
| CHETF2_ROOK (P) or ZHETF2_ROOK (P) | Computes the factorization of a complex Hermitian matrix using the bounded BunchKaufman ("rook") diagonal pivoting method (unblocked algorithm). |
| CHETRD or ZHETRD | Reduces a Hermitian matrix to a real symmetric tridiagonal form by using a unitary similarity transformation. |
| CHETRF (P) or ZHERTF (P) | Computes the factorization of a complex Hermitian-indefinite matrix using the diagonal pivoting method. |
| CHETRF_ROOK (P) or ZHERTF_ROOK (P) | Computes the factorization of a complex Hermitian-indefinite matrix using the BunchKaufman ("rook") diagonal pivoting method. |
| CHETRI (P) or ZHETRI (P) | Computes the inverse of a complex Hermitian indefinite matrix using the factorization computed by CHETRF or ZHETRF. |
| CHETRI_ROOK (P) or ZHETRI_ROOK (P) | Computes the inverse of a complex Hermitian indefinite matrix using the factorization computed by CHETRF_ROOK or ZHETRF_ROOK. |
| CHETRI2 or ZHETRI2 | Computes the inverse of a complex Hermitian-indefinite matrix using the factorization computed by CHETRF or ZHETRS. Sets the leading dimension of the workspace before calling CHETRI2X or ZHETRI2X that actually computes the inverse (extra precision). |
| CHETRI2X (P) or ZHETRI2X (P) | Computes the inverse of a complex Hermitian-indefinite matrix using the factorization computed by CHETRF or ZHETRS (extra precision). |
| CHETRS (P) or ZHETRS (P) | Solves a complex Hermitian-indefinite matrix using the factorization computed by CHETRF or ZHETRF. |
| CHETRS_ROOK (P) or ZHETRS_ROOK (P) | Solves a complex Hermitian-indefinite matrix using the factorization computed by CHETRF_ROOK or ZHETRF_ROOK. |
| CHETRS2 (P) or ZHETRS2 <br> (P) | Solves a system of linear equations with a complex Hermitian matrix using the factorization computed by CHETRF or ZHERTF and converted by CSYCONV or ZSYCONV. |
| CHFRK (P) or ZHFRK (P) | Performs a Hermitian rank-k operation for a matrix in the RFP format. |


| Routine | Function |
| :---: | :---: |
| CLA_HEAMV or ZLA_HEAMV | Performs a matrix-vector operation to calculate error bounds for a complex Hermitianindefinite matrix. |
| CLA_HERCOND_C (P) or ZLA_HERCOND_C (P) | Computes the infinity norm condition number of op(A)*inv(diag(c)) for a complex Hermitian-indefinite matrix. C is a REAL vector. |
| CLA_HERCOND_X (P) or ZLA_HERCOND_X (P) | Computes the infinity norm condition number of op(A)*inv(diag(x)) for a complex Hermitian-indefinite matrix. $X$ is a COMPLEX vector. |
| CLA_HERFSX_EXTENDED (P) or ZLA_HERFSX_EXTENDED (P) | Improves the computed solution to a system of linear equations for a complex Hermitian-indefinite matrix by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. |
| CLAHEF (P) or ZLAHEF (P) | Computes a partial factorization of a complex Hermitian-indefinite matrix, using the diagonal pivoting method. Used by CHETRF or CHETRF. |
| CLAHEF_ROOK (P) or ZLAHEF_ROOK (P) | Computes a partial factorization of a complex Hermitian-indefinite matrix, using the Bunch-Kaufman ("rook") diagonal pivoting method. Used by CHETRF_ROOK or CHETRF ROOK. |

TABLE 35 Hermitian Matrix in Packed Storage Routines

| Routine | Function |
| :--- | :--- |
| CHPCON or ZHPCON | Estimates the reciprocal of the condition number of a Hermitian-indefinite matrix in <br> packed storage using the factorization computed by CHPTRF or ZHPTRF. <br> Computes all the eigenvalues and eigenvectors of a Hermitian matrix in packed storage <br> (simple driver). Replacement with newer version CHPEVD or ZHPEVD suggested. <br> Computes selected eigenvalues and eigenvectors of a Hermitian matrix in packed <br> storage (expert driver). <br> CHPEVX (P) or ZHPEVX (P) |
| CHPEVD or ZHPEVD | Computes all the eigenvalues and eigenvectors of a Hermitian matrix in packed <br> storage, and uses a divide and conquer method to calculate eigenvectors (driver). <br> Reduces a Hermitian-definite generalized eigenproblem to standard form, where the <br> coefficient matrices are in packed storage, and uses the factorization computed by <br> CPPTRF or ZPPTRF. <br> Computes all the eigenvalues and eigenvectors of a generalized Hermitian-definite <br> eigenproblem where the coefficient matrices are in packed storage (simple driver). <br> Replacement with newer version CHPGVD or ZHPGVD suggested. <br> Computes all the eigenvalues and eigenvectors of a generalized Hermitian-definite <br> eigenproblem where the coefficient matrices are in packed storage, and uses a divide |
| and conquer method to calculate eigenvectors (driver). |  |
| CHPGVD or ZHPGVD | Computes selected eigenvalues and eigenvectors of a complex Hermitian-definite <br> eigenproblem, where the coefficient matrices are in packed storage (expert driver). |
| CHPRFS (P) or ZHPRFS (P) | Improves the computed solution to a system of linear equations when the coefficient <br> matrix is Hermitian indefinite in packed storage. <br> Computes the solution to a complex system of linear equations where the coefficient |
| CHatrix is a Hermitian matrix stored in the packed format (simple driver). |  |


| Routine | Function |
| :--- | :--- |
| CHPSVX or ZHPSVX | Uses the diagonal pivoting factorization to compute the solution to a complex system <br> of linear equations where the coefficient matrix is a Hermitian matrix stored in the <br> packed format (expert driver). <br> Reduces a complex Hermitian matrix stored in the packed form to a real symmetric <br> tridiagonal form by the unitary similarity transformation. <br> CHPTRD or ZHPTRD <br> CHPTRF or ZHPTRF <br> CHPTRI or ZHPTRI <br> Kaufman diagonal pivoting method. <br> Computes the inverse of a complex Hermitian-indefinite matrix in packed storage <br> using the factorization computed by CHPTRF or ZHPTRF. <br> Solves a complex Hermitian-indefinite matrix stored in the packed format using the <br> factorization computed by CHPTRF or ZHPTRF. |

table 36 Upper Hessenberg Matrix Routines

| Routine | Function |
| :--- | :--- |
| $x H S E I N(P)$ | Computes the specified right and/or left eigenvectors of an upper Hessenberg matrix <br> using inverse iteration. |
| CHSEQR or ZHSEQR | Computes the eigenvalues of a complex upper Hessenberg matrix and the Shur <br> factorization using the multishift QR algorithm. <br> SHSEQR (P) or DHSEQR (P) |
| Computes the eigenvalues of a real upper Hessenberg matrix and the Shur factorization <br> using the multishift QR algorithm. |  |

TABLE $37 \quad$ Upper Hessenberg Matrix-Generalized Problem (Hessenberg and Triangular Matrix) Routines

| Routine | Function |
| :--- | :--- |
| $x$ HGEQZ (P) | Computes the eigenvalues of a complex matrix pair (H,T), where H is an upper <br> Hessenberg matrix and T is an upper triangular, using the single/double-shift QZ <br> method. Matrix pairs of this type are produced by $x$ GGHRD. |

TABLE 38 Real Orthogonal Matrix in Packed Storage Routines

| Routine | Function |
| :--- | :--- |
| SOPGTR (P) or DOPGTR (P) | Generates an orthogonal transformation matrix from a real tridiagonal matrix <br> determined by SSPTRD or DSPTRD. |
| SOPMTR or DOPMTR | Multiplies a real general matrix by the orthogonal transformation matrix reduced to the <br> tridiagonal form by SSPTRD or DSPTRD. |

TABLE 39 Real Orthogonal Matrix Routines

| Routine | Function |
| :---: | :---: |
| SORBDB or DORBDB SORBDB1 or DORBDB1 | Simultaneously bidiagonalizes the blocks of a real partitioned orthogonal matrix. <br> Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 1). |
| SORBDB2 or DORBDB2 | Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 2). |
| SORBDB3 or DORBDB3 | Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 3). |
| SORBDB4 or DORBDB4 | Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 4). |
| SORBDB5 or DORBDB5 | Orthogonalizes the column vector X with respect to the orthonormal columns of Q . |
| SORBDB6 or DORBDB6 | Orthogonalizes the column vector X with respect to the orthonormal columns of Q . Used by SORBDB4 or DORBDB5. |
| SORG2L (P) or DORG2L (P) | Generates all or part of a real orthogonal matrix Q from a QL factorization, as determined by SGEQLF or DGEQLF (unblocked algorithm). |
| SORG2R (P) or DORG2R (P) | Generates all or part of a real orthogonal matrix Q from a QR factorization, as determined by SGEQRF or DGEQRF (unblocked algorithm). |
| SORGBR (P) or DORGBR | Generates the real orthogonal transformation matrices from reduction to the bidiagonal form, as determined by SGEBRD or DGEBRD. |
| SORGHR (P) or DORGHR (P) | Generates the real orthogonal transformation matrix reduced to the Hessenberg form, as determined by SGEHRD or DGEHRD. |
| SORGL2 (P) or DORGL2 (P) | Generates a real rectangular matrix with orthonormal rows, as returned by SGELQF or DGELQF. |
| SORGLQ (P) or DORGLQ (P) | Generates a real orthogonal matrix Q from an LQ factorization, as returned by SGELQF or DGELQF. |
| SORGQL (P) or DORGQL (P) | Generates a real orthogonal matrix Q from a QL factorization, as returned by SGEQLF or DGEQLF. |
| SORGQR (P) or DORGQR (P) | Generates a real orthogonal matrix Q from a QR factorization, as returned by SGEQRF or DGEQRF. |
| SORGR2 (P) or DORGR2 (P) | Generates all or part of a real orthogonal matrix Q from an RQ factorization determined SGEQRF or DGEQRF (unblocked algorithm). |
| SORGRQ (P) or DORGRQ (P) | Generates a real orthogonal matrix Q from an RQ factorization, as returned by SGERQF or DGERQF. |
| SORGTR (P) or DORGTR (P) | Generates a real orthogonal matrix reduced to tridiagonal form by SSYTRD or DSYTRD. |
| SORM22 or DORM22 | Multiplies a real general matrix by the orthogonal matrix. |
| SORM2L or DORM2L | Multiplies a real general matrix by the orthogonal matrix from a QL factorization determined by SGEQLF or DGEQLF (unblocked algorithm). |
| SORM2R or DORM2R | Multiplies a real general matrix by the orthogonal matrix from a QR factorization determined by SGEQRF or DGEQRF (unblocked algorithm). |
| SORMBR or DORMBR | Multiplies a real general matrix with the orthogonal matrix reduced to the bidiagonal form, as determined by SGEBRD or DGEBRD. |


| Routine | Function |
| :--- | :--- |
| SORMHR or DORMHR | Multiplies a real general matrix by the orthogonal matrix reduced to the Hessenberg <br> form by SGEHRD or DGEHRD. <br> SORML2 or DORML2 <br> Multiplies a real general matrix by the orthogonal matrix from an LQ factorization <br> determined by SGELQF (unblocked algorithm). <br> Multiplies a real general matrix by the orthogonal matrix from an LQ factorization, as <br> returned by SGELQF or DGELQF. <br> Multiplies a real general matrix by the orthogonal matrix from a QL factorization, as <br> returned by SGEQLF or DGEQLF. <br> SORMQL or DORMQL |
| SORMQR or DORMQR | Multiplies a real general matrix by the orthogonal matrix from a QR factorization, as <br> returned by SGEQRF or DGEQRF. <br> SORMR or DORMR2 |
| SORMR3 or DORMR3 |  |
| determined by STZRZF or DTZRZF (unblocked algorithm). |  |
| SORMRQ or DORMRQ | Multiplies a real general matrix by the orthogonal matrix from an RZ factorization <br> determined by STZRZF or DTZRZF (unblocked algorithm). <br> Multiplies a real general matrix by the orthogonal matrix from an RQ factorization |
| returned by SGERQF or DGERQF. |  |
| Multiplies a real general matrix by the orthogonal matrix from an RZ factorization, as |  |

TABLE 40 Symmetric or Hermitian Positive Definite Band Matrix Routines

| Routine | Function |
| :---: | :---: |
| xPBCON | Estimates the reciprocal of the condition number of a symmetric or Hermitian positive definite band matrix using the Cholesky factorization returned by $x$ PBTRF. |
| $x$ PBEQU (P) | Computes equilibration scale factors for a symmetric or Hermitian positive definite band matrix. |
| $x$ PBRFS (P) | Refines solution to a symmetric or Hermitian positive definite banded system of linear equations. |
| $x$ PBSTF | Computes a split Cholesky factorization of a real symmetric positive definite band matrix. |
| $x$ PBSV | Solves a symmetric or Hermitian positive definite banded system of linear equations (simple driver). |
| $x$ PBSVX (P) | Solves a symmetric or Hermitian positive definite banded system of linear equations (expert driver). |
| xPBTF2 | Computes the Cholesky factorization of a real symmetric or complex Hermitian positive definite band matrix (unblocked algorithm). |
| xPBTRF | Computes the Cholesky factorization of a symmetric or Hermitian positive definite band matrix. |
| $x$ PBTRS | Solves a system of linear equations with a real symmetric or complex Hermitian positive definite banded matrix using the Cholesky factorization computed by $x$ PBTRF. |

TABLE 41 Symmetric or Hermitian Positive Definite Matrix Routines

| Routine | Function |
| :---: | :---: |
| CLA_PORCOND_C (P) or | Computes the infinity norm condition number of op(A)*inv(diag (c)) for a complex |
| ZLA_PORCOND_C (P) | Hermitian positive definite matrix. C is a REAL vector. |
| CLA_PORCOND_X (P) or | Computes the infinity norm condition number of op(A)*inv( $\operatorname{diag}(x)$ ) for a complex |
| ZLA_PORCOND_X (P) | Hermitian positive definite matrix. X is a COMPLEX vector. |
| SLA_PORCOND (P) or | Estimates the Skeel condition number for a real symmetric positive definite matrix. |
| DLA_PORCOND(P) |  |
| $x$ LA_LIN_BERR (P) | Computes a component-wise relative backward error. |
| $x$ LA_PORFSX_EXTENDED (P) | Improves the computed solution to a system of linear equations for a real symmetric or complex Hermitian positive definite matrix by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. |
| $x$ LA_WWADDW | Adds a vector W into a doubled-single vector (X, Y). This works for all extant IBM's hex and binary floating point arithmetics, but not for decimal. |
| $x$ PFTRF | Computes the Cholesky factorization of a real symmetric or Hermitian positive definite band matrix. |
| $x$ PFTRI | Computes the inverse of a real symmetric or Hermitian positive definite matrix, using the Cholesky factorization computed by xPFTRF. |
| $x$ PFTRS | Solves a system of linear equations with a symmetric or Hermitian positive definite matrix, using the Cholesky factorization computed by xPFTRF. |
| xPOCON | Estimates the reciprocal of the condition number of a symmetric or Hermitian positive definite matrix, using the Cholesky factorization returned by xPOTRF. |
| $x$ POEQU (P) | Computes equilibration scale factors for a symmetric or Hermitian positive definite matrix. |
| $x$ POEQUB (P) | Computes row and column scalings intended to equilibrate a symmetric or Hermitian positive definite matrix and reduce its condition number with respect to the two-norm. |
| $x$ PORFS (P) | Refines the solution to a linear system in a Cholesky-factored symmetric or Hermitian positive definite matrix. |
| $x$ PORFSX (P) | Improves the computed solution to a system of linear equations, when the coefficient matrix is a real symmetric or Hermitian positive definite, and provides the error bounds and backward-error estimates for the solution (extra precision). |
| xPOSV | Solves a symmetric or Hermitian positive definite system of linear equations (simple driver). |
| $x$ POSVX (P) | Solves a symmetric or Hermitian positive definite system of linear equations (expert driver). |
| $x$ POSVXX (P) | Solves a real symmetric or Hermitian positive definite system of linear equations (expert driver, extra precision). If requested, both normwise and maximum componentwise error bounds are returned. |
| $x$ POTRF | Computes the Cholesky factorization of a real symmetric or Hermitian positive definite matrix. |
| $x$ POTRF2 | Computes the Cholesky factorization of a real symmetric or Hermitian positive definite matrix using the recursive algorithm. |
| $x$ POTRI | Computes the inverse of a real symmetric or Hermitian positive definite matrix using the Cholesky-factorization computed by $x$ POTRF. |


| Routine | Function |
| :--- | :--- |
| $x$ POTRS | Solves a real symmetric or Hermitian positive definite system of linear equations, using <br> the Cholesky factorization computed by $x$ POTRF. |
| ZCPOSV | Computes the solution to a complex system of linear equations with a positive definite <br> matrix (mixed precision with iterative refinement). |

TABLE 42 Symmetric or Hermitian Positive Definite Matrix in Packed Storage Routines

| Routine | Function |
| :---: | :---: |
| xPPCON | Estimates the reciprocal of the condition number of a Cholesky-factored symmetric positive definite matrix in packed storage. |
| $x$ PPEQU (P) | Computes equilibration scale factors for a symmetric or Hermitian positive definite matrix in packed storage. |
| $x$ PPRFS (P) | Refines the solution to a linear system of equations in a Cholesky-factored symmetric or Hermitian positive definite matrix in packed storage. |
| $x$ PPSV | Solves a linear system in a symmetric or Hermitian positive definite matrix in packed storage (simple driver). |
| $x$ PPSVX (P) | Solves a linear system in a symmetric or Hermitian positive definite matrix in packed storage (expert driver). |
| xPPTRF | Computes the Cholesky factorization of a real symmetric or Hermitian positive definite matrix stored in the packed format. |
| $x$ PPTRI | Computes the inverse of a real symmetric or Hermitian positive definite matrix in packed storage using the Cholesky factorization returned by xPPTRF. |
| $x$ PPTRS | Solves a real symmetric or Hermitian positive definite system of linear equations where the coefficient matrix is in packed storage, using the Cholesky factorization returned by xPPTRF. |
| xPSTF2 (P) | Computes the Cholesky factorization with complete pivoting of a real symmetric or Hermitian positive-semi-definite matrix. This version of the algorithm calls level 2 BLAS. |
| $x$ PSTRF (P) | Computes the Cholesky factorization with complete pivoting of a real symmetric or Hermitian positive-semi-definite matrix. This version of the algorithm calls level 3 BLAS. |

TABLE 43 Symmetric or Hermitian Positive Definite Tridiagonal Matrix Routines

| Routine | Function |
| :--- | :--- |
| $x$ PTCON | Estimates the reciprocal of the condition number of a real symmetric or Hermitian <br> positive definite tridiagonal matrix using the Cholesky factorization computed by <br> $x$ PTTRF. |
| $x$ CTEQR (P) | Computes all the eigenvectors and, optionally, the eigenvalues of a real symmetric or <br> Hermitian positive definite matrix. <br> Refines the solution to a symmetric or Hermitian positive definite tridiagonal system of <br> linear equations. |


| Routine | Function |
| :---: | :---: |
| xPTSV | Solves a real symmetric or Hermitian positive definite tridiagonal system of linear equations (simple driver). |
| xPTSVX | Solves a real symmetric or Hermitian positive definite tridiagonal system of linear equations (expert driver). |
| $x$ PTTRF | Computes the LDL ${ }^{\mathrm{H}}$ or LDL ${ }^{\mathrm{T}}$ factorization of a real symmetric or Hermitian positive definite tridiagonal matrix. |
| $x$ PTTRS | Solves a real symmetric or Hermitian positive definite tridiagonal system of linear equations using the $\operatorname{LDL}^{\mathrm{H}}$ or LDL ${ }^{\mathrm{T}}$ factorization returned by $x$ PTTRF. |
| xPTTS2 (P) | Solves a tridiagonal system of linear equations using the LDL ${ }^{\mathrm{H}}$ or $\mathrm{LDL}^{\mathrm{T}}$ factorization computed by $x$ PTTRF. Used by $x$ PTTRS. |

TABLE 44 Real Symmetric Band Matrix Routines

| Routine | Function |
| :--- | :--- |
| SSBEV or DSBEV | Computes all the eigenvalues and, optionally, the left and/or right eigenvectors of a real <br> symmetric band matrix (simple driver). Replacement with newer version SSBEVD or <br> DSBEVD suggested. <br> Computes all the eigenvalues and, optionally, the eigenvectors of a real symmetric <br> band matrix. If eigenvectors are desired, it uses a divide and conquer algorithm. <br> (driver) <br> SSBEVD or DSBEVD |
| SSBEVX (P) or DSBEVX (P) | Computes selected eigenvalues and, optionally, the left and/or right eigenvectors of a <br> symmetric band matrix (expert driver). <br> Reduces a symmetric-definite banded generalized eigenproblem to a standard form. |
| SSBGST (P) or DSBGST (P) | Computes all the eigenvalues and, optionally, the eigenvectors of a generalized <br> symmetric-definite banded eigenproblem (simple driver). Replacement with newer <br> version SSBGVD or DSBGVD suggested. <br> Computes all the eigenvalues and, optionally, the eigenvectors of generalized <br> symmetric-definite banded eigenproblem and uses a divide and conquer method to <br> calculate eigenvectors (simple driver). <br> Computes selected eigenvalues and eigenvectors of a generalized symmetric-definite |
| SSBGVX (P) or DSBGVX (P) | Canded eigenproblem (expert driver). |
| SSBTRD (P) or DSBTRD (P) | Reduces a symmetric band matrix to real symmetric tridiagonal form by using an <br> orthogonal similarity transformation. |

TABLE 45 Symmetric Matrix in Packed Storage Routines

| Routine | Function |
| :--- | :--- |
| $x$ SPCON | Estimates the reciprocal of the condition number of a real or complex symmetric <br> packed matrix using the factorization computed by $x$ SPTRF. |
| SSFRK (P) or DSFRK (P) | Performs a symmetric rank-k operation for a real matrix in RFP format. <br> Computes all the eigenvalues and eigenvectors of a symmetric matrix in packed storage <br> (simple driver). Replacement with newer version SSPEVD or DSPEVD suggested. |


| Routine | Function |
| :---: | :---: |
| SSPEVD or DSPEVD | Computes all the eigenvalues and, optionally, the light and/or right eigenvectors of a symmetric matrix in packed storage. If eigenvectors are desired, it uses a divide and conquer algorithm (simple driver). |
| SSPEVX (P) or DSPEVX (P) | Computes selected eigenvalues and eigenvectors of a symmetric matrix in packed storage (expert driver). |
| SSPGST or DSPGST | Reduces a real symmetric-definite generalized eigenproblem to a standard form where the coefficient matrices are in packed storage and uses the factorization computed by SPPTRF or DPPTRF. Replacement with newer version SSPGVD or DSPGVD suggested. |
| SSPGV or DSPGV | Computes all the eigenvalues and eigenvectors of a real generalized symmetric-definite eigenproblem where the coefficient matrices are in packed storage (simple driver). Replacement with newer version SSPGVD or DSPGVD suggested. |
| SSPGVD or DSPGVD | Computes all the eigenvalues and eigenvectors of a real generalized symmetric-definite eigenproblem where the coefficient matrices are in packed storage, and uses a divide and conquer method to calculate eigenvectors (driver). |
| SSPGVX or DSPGVX | Computes selected eigenvalues and eigenvectors of a real generalized symmetricdefinite eigenproblem where the coefficient matrices are in packed storage (expert driver). |
| DSPOSV | Computes the solution to a real system of linear equations with a real symmetric positive definite matrix: first attempts to factorize the matrix in single precision, then, if necessary - with double precision. |
| $x$ SPRFS (P) | Improves the computed solution to a real or complex system of linear equations when the coefficient matrix is symmetric indefinite in packed storage. |
| $x$ SPSV | Computes the solution to a real or complex system of linear equations where the coefficient matrix is a symmetric matrix in packed storage (simple driver). |
| $x$ SPSVX | Uses the diagonal pivoting factorization to compute the solution to a system of linear equations where the coefficient matrix is a symmetric matrix in packed storage (expert driver). |
| SSPTRD or DSPTRD | Reduces a real symmetric matrix stored in the packed form to a real symmetric tridiagonal form using an orthogonal similarity transformation. |
| $x$ SPTRF | Computes the factorization of a symmetric packed matrix using the Bunch-Kaufman diagonal pivoting method. |
| $x$ SPTRI | Computes the inverse of a symmetric indefinite matrix in packed storage using the factorization computed by $x$ SPTRF. |
| $x$ SPTRS (P) | Solves a system of linear equations with a real or complex symmetric matrix in packed storage using the factorization computed by $x$ SPTRF. |

TABLE 46 Real Symmetric Tridiagonal Matrix Routines

| Routine | Function |
| :--- | :--- |
| $x$ LAED0 (P) | Computes all the eigenvalues and corresponding eigenvectors of a real or complex <br> unreduced symmetric tridiagonal matrix using the divide and conquer method. Used by <br> $x$ STEDC. |


| Routine | Function |
| :---: | :---: |
| SLAED1 (P) or DLAED1 (P) | Computes the updated eigensystem of a real diagonal matrix after modification by a rank-one symmetric matrix. Used by SSTEDC or DSTEDC, when the original matrix is tridiagonal. |
| SLAED2 (P) or DLAED2 (P) | Merges the two sets of eigenvalues together into a single sorted set and tries to deflate the size of the problem. Used by SSTEDC or DSTEDC. |
| SLAED3 (P) or DLAED3 | Finds the roots of the secular equation and updates the eigenvectors. Used by SSTEDC or DSTEDC, when the original matrix is tridiagonal. |
| SLAED4 (P) or DLAED4 (P) | Finds a single root of the secular equation. Used by SSTEDC or DSTEDC. |
| SLAED5 or DLAED5 | Solves a 2-by-2 secular equation. Used by SSTEDC or DSTEDC. |
| SLAED6 or DLAED6 | Computes the positive or negative root closest to the origin (one Newton step in solution of the secular equation). |
| $x$ LAED7 (P) | Computes the updated eigensystem of a diagonal matrix after modification by a rankone symmetric matrix. Used by $x$ STEDC, when the original matrix is dense. |
| $x$ LAED8 (P) | Merges the two sets of eigenvalues into a single sorted set and deflates the secular equation. Used by $x$ STEDC, when the original matrix is dense. |
| SLAED9 (P) or DLAED9 (P) | Finds the roots of the secular equation and updates the eigenvectors. Used by SSTEDC or DSTEDC, when the original matrix is dense. |
| SLAEDA (P) or DLAEDA (P) | Computes a vector determining the rank-one modification of the diagonal matrix. Used by SSTEDC or DSTEDC, when the original matrix is dense. |
| SLAGTF or DLAGTF (P) | Computes an LU factorization of a matrix T - (lambda * I), where T is a general tridiagonal matrix, and lambda is a scalar, using partial pivoting with row interchanges. Used by SSTEIN or DSTEIN. |
| SSTEBZ or DSTEBZ | Computes the eigenvalues of a real symmetric tridiagonal matrix. |
| CSTEDC (P) or ZSTEDC (P) | Computes all the eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band complex Hermitian matrix can also be found if CHETRD/ZHETRD, CHPTRD/ZHPTRD, or CHBTRD/ZHBTRD has been used to reduce this matrix to tridiagonal form. |
| SSTEDC or DSTEDC | Computes all the eigenvalues and eigenvectors of a complex symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band real symmetric matrix can also be found if SSYTRD, SSPTRD, or SSBTRD; or DSYTRD, DSPTRD, or DSBTRD has been used to reduce this matrix to tridiagonal form. |
| $x$ STEGR | Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix using Relatively Robust Representations, $x$ STEGR is a compatibility wrapper around the improved $x$ STEMR routine. |
| $x$ STEIN (P) | Computes selected eigenvectors of a real symmetric tridiagonal matrix using inverse iteration. |
| $x$ STEMR (P) | Computes the selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix using Relatively Robust Representations. |
| $x$ STEQR (P) | Computes all the eigenvalues and eigenvectors of a real symmetric tridiagonal matrix using the Pal-Walker-Kahan variant of a QL or QR algorithm. |
| SSTERF (P) or DSTERF (P) | Computes all the eigenvalues and eigenvectors of a real symmetric tridiagonal matrix using a root-free QL or QR algorithm variant. |


| Routine | Function |
| :--- | :--- |
| SSTEV or DSTEV | Computes all the eigenvalues and eigenvectors of a real symmetric tridiagonal matrix <br> (simple driver). Replacement with newer version SSTEVR or DSTEVR suggested. <br> Computes all the eigenvalues and eigenvectors of a real symmetric tridiagonal matrix <br> (simple driver). Replacement with newer version SSTEVR or DSTEVR suggested. <br> Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix <br> using Relatively Robust Representations. <br> SSTEVR or DSTEVR <br> SSTEVX (P) or DSTEVX (P) <br> $x$ Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix <br> (expert driver). <br> Computes the solution to a system of linear equations where the coefficient matrix is a <br> symmetric tridiagonal matrix (unblocked algorithm). <br> Computes the factorization of a real or complex symmetric tridiagonal matrix using the <br> Bunch-Kaufman diagonal pivoting method (unblocked algorithm). |

TABLE 47 Symmetric Matrix Routines

| Routine | Function |
| :---: | :---: |
| $x$ LA_SYAMV | Performs a matrix-vector operation to calculate error bounds for a real or complex symmetric indefinite matrix. |
| CLA_SYRCOND_C (P) or ZLA_SYRCOND_C (P) | Computes the infinity norm condition number of op(A)*inv(diag(c)) for a real or complex symmetric indefinite matrix. C is a REAL vector. |
| CLA_SYRCOND_X (P) or ZLA_SYRCOND_X (P) | Computes the infinity norm condition number of $o p(A) * \operatorname{inv}(\operatorname{diag}(x))$ for a real or complex symmetric indefinite matrix. X is a COMPLEX vector. |
| SLA_SYRCOND (P) or DLA_SYRCOND(P) | Estimates the Skeel condition number for a real symmetric indefinite matrix. |
| xLA_SYRFSX_EXTENDED(P) | Improves the computed solution to a system of linear equations of a real or complex symmetric indefinite matrix by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. |
| XLASYF | Computes a partial factorization of a real or complex symmetric matrix, using the diagonal pivoting method. Used by $x$ SYTRF. |
| $x$ LASYF_ROOK | Computes a partial factorization of a real or complex symmetric matrix, using the bounded Bunch-Kaufman ("rook") diagonal pivoting method. Used by xSYTRF_ROOK. |
| $x$ SYCON | Estimates the reciprocal of the condition number of a real or complex symmetric matrix using the factorization computed by $x$ SYTRF. |
| xSYCON_ROOK | Estimates the reciprocal of the condition number of a real or complex symmetric matrix using the factorization computed by $x$ SYTRF_ROOK. |
| $x$ SYCONV (P) | Converts the matrix computed by SSYTRF or DSYTRF into lower and upper triangular matrices and vice-versa. |
| $x$ SYEQUB (P) | Computes row and column scalings intended to equilibrate a real or complex symmetric matrix and reduce its condition number with respect to the two-norm. |
| SSYEV or DSYEV | Computes all eigenvalues and eigenvectors of a symmetric matrix (simple driver). Replacement with newer version SSYEVR or DSYEVR suggested. |


| Routine | Function |
| :---: | :---: |
| SSYEVD or DSYEVD | Computes all eigenvalues and eigenvectors of a symmetric matrix and uses a divide and conquer method to calculate eigenvectors (expert driver). Replacement with newer version SSYEVR or DSYEVR suggested. |
| SSYEVR or DSYEVR | Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix. |
| SSYEVX (P) or DSYEVX (P) | Computes eigenvalues and eigenvectors of a real symmetric matrix (expert driver). |
| SSYGS2 or DSYGS2 | Reduces a real symmetric-definite generalized eigenproblem to a standard form using the factorization results obtained from SPOTRF or DPOTRF (unblocked algorithm). |
| SSYGST or DSYGST | Reduces a symmetric-definite generalized eigenproblem to standard form using the factorization computed by SPOTRF or DPOTRF. |
| SSYGV or DSYGV | Computes all the eigenvalues and eigenvectors of a generalized symmetric-definite eigenproblem. Replacement with newer version SSYGVD or DSYGVD suggested. |
| SSYGVD or DSYGVD | Computes all the eigenvalues and eigenvectors of a generalized symmetric-definite eigenproblem and uses a divide and conquer method to calculate eigenvectors (driver). |
| SSYGVX or DSYGVX | Computes selected eigenvalues and eigenvectors of a generalized symmetric-definite eigenproblem (expert driver). |
| $x$ SYRFS (P) | Improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite. |
| $x$ SYRFSX (P) | Improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and provides error bounds and backward error estimates for the solution (extra precision). |
| $x$ SYSV | Solves a real or complex symmetric indefinite system of linear equations (simple driver). $x$ SYTRF is called to compute the factorization of a complex symmetric matrix using the diagonal pivoting method. |
| xSYSV_ROOK | Solves a real or complex symmetric indefinite system of linear equations (simple driver). $x$ SYTRF_ROOK is called to compute the factorization of a complex symmetric matrix using the bounded Bunch-Kauffman ("rook") diagonal pivoting method. |
| $x$ SYSVX | Solves a real or complex symmetric indefinite system of linear equations (expert driver). |
| $x$ SYSVXX (P) | Solves a real or complex symmetric indefinite system of linear equations (expert driver, extra precision). If requested, both normwise and maximum component-wise error bounds are returned. |
| SSYTD2 or DSYTD2 | Reduces a real symmetric matrix to a real symmetric tridiagonal form by an orthogonal similarity transformation (unblocked algorithm). |
| xSYTF2 | Computes the factorization of a real or complex symmetric indefinite matrix, using the diagonal pivoting method (unblocked algorithm). |
| xSYTF2_ROOK | Computes the factorization of a real or complex symmetric indefinite matrix, using the bounded Bunch-Kauffman ("rook") diagonal pivoting method (unblocked algorithm). |
| SSYTRD or DSYTRD | Reduces a real symmetric matrix to a real symmetric tridiagonal form by using an orthogonal similarity transformation. |
| $x$ SYTRF (P) | Computes the factorization of a real or complex symmetric indefinite matrix using the Bunch-Kaufman diagonal pivoting method (blocked algorithm). |


| Routine | Function |
| :--- | :--- |
| $x$ SYTRI | Computes the inverse of a real or complex symmetric indefinite matrix using the <br> factorization computed by $x$ SYTRF. <br> Computes the inverse of a real or complex symmetric indefinite matrix using the <br> factorization computed by $x$ SYTRF_ROOK. |
| $x$ SYTRI2 | Computes the inverse of a real or complex symmetric indefinite matrix using the <br> factorization computed by $x$ SYTRF. Sets the leading dimension of the workspace before <br> calling $x$ SYTRF2X that actually computes the inverse. <br> Computes the inverse of a real or complex symmetric indefinite matrix using the <br> factorization computed by $x$ SYTRF. Used by $x$ SYTRI2. <br> Solves a system of linear equations with a real or complex symmetric matrix using the <br> factorization computed by $x$ SYTRF. <br> $x$ SYTRS (P) |
| $x$ Solves a system of linear equations with a real or complex symmetric matrix using the |  |
| factorization computed by $x$ SYTRF_ROOK. |  |

table 48 Triangular Band Matrix Routines

| Routine | Function |
| :--- | :--- |
| $x$ TBCON | Estimates the reciprocal of the condition number of a triangular band matrix. <br> $x$ Determines error bounds and estimates for solving a triangular banded system of linear <br> equations. <br> Solves a triangular banded system of linear equations. |
| $x$ TBTRS |  |

TABLE 49 Triangular Matrix-Generalized Problem (Pair of Triangular Matrices) Routines

| Routine | Function |
| :--- | :--- |
| $x$ TGEVC (P) | Computes some or all of the right and/or left eigenvectors of a pair of real or complex <br> triangular matrices, computed by $x$ GGGRD and $x$ HGEQZ. <br> Reorders the generalized Schur decomposition of a real or complex matrix pair using <br> an orthogonal or unitary equivalence transformation. <br> Reorders the generalized Schur decomposition of a real or complex matrix pair and <br> computes the generalized eigenvalues. <br> Computes the generalized singular value decomposition (SVD) from two real or <br> complex triangular or trapezoidal matrices obtained from $x G G S V P$. <br> Estimates the reciprocal of the condition numbers for specified eigenvalues and <br> eigenvectors of two matrices in generalized Schur canonical form. <br> Estimates the reciprocal of the condition numbers for specified eigenvalues and |
| CTGSNA (P) or ZTGSNA (P) | Estimat <br> eigenvectors of two matrices in generalized real Schur canonical form. <br> Solves the generalized Sylvester equation. |
| $x$ STGSYL |  |

TABLE 50 Triangular Matrix in Packed Storage Routines

| Routine | Function |
| :--- | :--- |
| $x$ TPCON | Estimates the reciprocal or the condition number of a triangular matrix in packed <br> storage. <br> Applies a real or complex orthogonal matrix obtained from a "triangular-pentagonal" <br> block reflector to a general matrix, which consists of two blocks. <br> Computes a blocked QR factorization of a real or complex "triangular-pentagonal" <br> matrix, which is composed of a triangular block and a pentagonal block, using the <br> compact WY representation. <br> Computes a QR factorization of a real or complex "triangular-pentagonal" matrix, <br> which is composed of a triangular block and a pentagonal block, using the compact |
| $x$ TPQRT | WY representation. <br> Provides error bounds and backward error estimates for the solution to a real or <br> complex system of linear equations with a triangular packed coefficient matrix. The <br> solution should be preliminary obtained by $x$ TPTRS or some other means. <br> Computes the inverse of a real or complex triangular matrix in packed storage. |
| $x$ TPRFS (P) | Solves a real or complex triangular system of linear equations where the coefficient <br> matrix is in packed storage. <br> Copies a real or complex triangular matrix from the standard packed format (TP) to the |
| $x$ TPTRI | rectangular full packed format (TF). <br> $x$ Copies a real or complex triangular matrix from the standard packed format (TP) to the <br> standard full-packed format (TR). |
| $x$ TPTTR |  |

TABLE 51 Triangular Matrix in Rectangular Full-Packed (RFP) Format and Standard Packed Format Routines
\(\left.$$
\begin{array}{l|l}\hline \text { Routine } & \text { Function } \\
\hline x \text { TFSM (P) } & \begin{array}{l}\text { Solves a matrix equation with real or complex matrices. One operand is a triangular } \\
\text { matrix in the RFP format. } \\
\text { Computes the inverse of a real or complex triangular matrix stored in RFP format. } \\
\text { Copies a real or complex triangular matrix from the rectangular full-packed format } \\
\text { (TF) to the standard packed format (TP). } \\
x \text { TFTRI } \\
x \text { TFTTP } \\
x \text { TFTTR }\end{array}
$$ <br>
x (TF) to the standard full format (TR). <br>
x Copies a real or complex triangular matrix from the standard packed format (TP) to the <br>
rectangular full packed format (TF). <br>
Copies a real or complex triangular matrix from the standard packed format (TP) to the <br>
standard full-packed format (TR). <br>

Copies a real or complex triangular matrix from the standard full format (TR) to the\end{array}\right\}\)| rectangular full-packed format (TF). |
| :--- |
| $x$ Copies a real or complex triangular matrix from the standard full format (TR) to the |
| standard packed format (TP). |

TABLE 52 Triangular Matrix Routines

| Routine | Function |
| :---: | :---: |
| xTRCON | Estimates the reciprocal or the condition number of a real or complex triangular matrix. |
| $x$ TREVC (P) | Computes right and/or left eigenvectors of a real or complex upper triangular matrix. |
| $x$ TREVC3 (P) | Computes some or all right and/or left eigenvectors of a real or complex upper quasitriangular matrix. |
| $x$ TREXC | Reorders the Schur factorization of a real or complex matrix using an orthogonal or unitary similarity transformation. |
| $x$ TRRFS (P) | Provides error bounds and estimates for a triangular system of linear equations with a real or complex triangular matrix. |
| CTRSEN (P) or ZTRSEN (P) | Reorders the Schur factorization of a complex matrix $\mathrm{A}=\mathrm{Q}^{*} \mathrm{~T}^{*} \mathrm{Q}^{* *} \mathrm{H}$, so that a selected cluster of eigenvalues appears in the leading positions in the diagonal of the upper triangular matrix T , and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace. |
| STRSEN or DTRSEN | Reorders the real Schur factorization of a real matrix $A=Q^{*} T^{*} Q^{* *} T$, so that a selected cluster of eigenvalues appears in the leading positions in the diagonal of the upper triangular matrix T , and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace. |
| $\chi$ TRSNA (P) | Estimates the reciprocal condition numbers of selected eigenvalues and eigenvectors of an upper quasi-triangular matrix. |
| $x$ TRSYL | Solves a Sylvester matrix equation. |
| $x$ TRTRI | Computes the inverse of a real or complex triangular matrix (unblocked algorithm). |
| $x$ TRTRS | Solves a triangular system of linear equations. |

TABLE 53 Trapezoidal Matrix Routines

| Routine | Function |
| :--- | :--- |
| $x$ LARZ | Applies an elementary reflector (as returned by $x$ TZRZF) to a real or complex general <br> matrix. <br> $x$ LARZB (P) <br> Applies a block reflector or its transpose to a real general matrix or applies a block <br> reflector or its conjugate-transpose to a complex general matrix. |
| $x$ FARZT | Forms the triangular factor T of a real or complex block reflector H, which is defined <br> as a product of k elementary reflectors. <br> Deprecated routine replaced by $x$ ORMZ. Applies a Householder matrix generated by <br> $x$ xZRQF to a real or complex matrix. |
| $x$ DCATZM | Deprecated routine replaced by routine $x$ TZRZF. <br> Reduces a rectangular upper trapezoidal matrix to an upper triangular form by means <br> of orthogonal transformations. |
| $x$ TZRZF (P) |  |

TABLE 54 Unitary Matrix Routines

| Routine | Function |
| :---: | :---: |
| CUNBDB or ZUNBDB CUNBDB1 or ZUNBDB1 | Simultaneously bidiagonalizes the blocks of an M-by-M partitioned unitary matrix. <br> Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 1). |
| CUNBDB2 or ZunbdB2 | Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 2). |
| CUNBDB3 or ZUNBDB3 | Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 3). |
| CUNBDB4 or ZunbdB4 | Simultaneously bidiagonalizes the blocks of a tall and skinny matrix with orthonormal columns (variant 4). |
| CUNBDB5 or ZunbdB5 | Orthogonalizes the column vector X with respect to the orthonormal columns of Q . |
| CUNBDB5 or ZunbdB5 | Orthogonalizes the column vector X with respect to the orthonormal columns of Q . Used by CUNBDB5 or ZUNBDB5. |
| CUNCSD2BY1 or ZUNCSD2BY1 | Computes the CS decomposition of an M-by-Q matrix X with orthonormal columns that has been partitioned into a 2-by-1 block structure. |
| CUNG2L (P) or Zung2L (P) | Generates an M-by-N complex matrix Q with orthonormal columns, which is defined as the last N columns of a product of K elementary reflectors of order M , as returned by CGEQLF or ZGEQLF. |
| CUNG2R (P) or ZUNG2R (P) | Generates an M-by-N complex matrix Q with orthonormal columns, which is defined as the last N columns of a product of K elementary reflectors of order M , as returned by CGEQRF or ZGEQRF. |
| CUNGBR (P) or Zungbr (P) | Generates an unitary transformation matrix from reduction to bidiagonal form, as determined by CGEBRD or ZGEBRD. |
| CUNGHR (P) or Zunghr (P) | Generates an orthogonal transformation matrix reduced to Hessenberg form, as determined by CGEHRD or ZGEHRD. |
| CUNGL2 (P) or ZUNGL2 (P) | Generates all or part of an unitary matrix Q from an LQ factorization determined by CGELQF or ZGELQF (unblocked algorithm). |
| CUNGLQ (P) or ZungLQ (P) | Generates an unitary matrix Q from an LQ factorization, as returned by CGELQF or ZGELQF. |
| CUNGQL (P) or ZUNGQL (P) | Generates an unitary matrix Q from a QL factorization, as returned by CGEQLF or ZGEQLF. |
| CUNGQR (P) or ZUNGQR (P) | Generates an unitary matrix Q from a QR factorization, as returned by CGEQRF or ZGEQRF. |
| CUNGR2 (P) or ZUNGR2 (P) | Generates all or part of an unitary matrix Q from an RQ factorization determined by CGERQF or ZGERQF (unblocked algorithm). |
| CUNGRQ (P) or ZUNGRQ (P) | Generates an unitary matrix Q from an RQ factorization, as returned by CGERQF or ZGERQF. |
| CUNGTR (P) or ZungTr (P) | Generates an unitary matrix reduced to a tridiagonal form, by CHETRD or ZHETRD. |
| CUNM22 or Zunm22 | Multiplies a general matrix by a banded unitary matrix. |
| CUNM2L or ZUNM2L | Multiplies a general matrix by the unitary matrix from a QL factorization determined by CGEQLF or ZGEQLF (unblocked algorithm). |


| Routine | Function |
| :--- | :--- |
| CUNM2R or ZUNM2R | Multiplies a general matrix by an unitary matrix from a QR factorization determined by <br> CGEQRF or ZGERLF (unblocked algorithm). |
| CUNMBR or ZUNMBR | Multiplies a general matrix with an unitary transformation matrix reduced to a <br> bidiagonal form, as determined by CGEBRD or ZGEBRD. <br> Multiplies a general matrix by an unitary matrix reduced to the Hessenberg form by <br> CGEHRD or ZGEHRD. <br> CUNMHR or ZUNMHR |
| CUNML2 or ZUNML2 | Multiplies a general matrix by an unitary matrix from an LQ factorization determined <br> by CGELQF or ZGELQF (unblocked algorithm). <br> Multiplies a general matrix by an unitary matrix from an LQ factorization, as returned <br> by CGELQF or ZGELQF. <br> CUNMLQ or ZUNMLQ |
| CUNMQR or ZUNMQR | Multiplies a general matrix by an unitary matrix from a QL factorization, as returned <br> by CGEQLF or ZGEQLF. <br> Multiplies a general matrix by an unitary matrix from a QR factorization, as returned <br> by CGEQRF or ZGEQRF. <br> CUltiplies a general matrix by an unitary matrix from an RQ factorization determined <br> by CGERQF or ZGERQF (unblocked algorithm). |
| CUNMR3 or ZUNMR3 | Multiplies a general matrix by an unitary matrix from an RZ factorization determined <br> by CTZRZF or ZTZRZF (unblocked algorithm). |
| CUNMRQ or ZUNMRQ | Multiplies a general matrix by an unitary matrix from an RQ factorization, as returned <br> by CGERQF or ZGERQF. |
| CUNMTR or ZUNMTR | Multiplies a general matrix by an unitary matrix from an RZ factorization, as returned <br> by CTZRZF or ZTZRZF. |
| Multiplies a general matrix by an unitary transformation matrix reduced to tridiagonal |  |
| form by CHETRD or ZHETRD. |  |

TABLE $55 \quad$ Unitary Matrix in Packed Storage Routines

| Routine | Function |
| :--- | :--- |
| CUPGTR (P) or ZUPGTR (P) | Generates an unitary transformation matrix from a tridiagonal matrix determined by <br> CHPTRD or ZHPTRD. |
| CUPMTR or ZUPMTR | Multiplies a general matrix by an unitary transformation matrix reduced to tridiagonal <br> form by CHPTRD or ZHPTRD. |

## BLAS1 Routines

Table 56, "BLAS1 (Basic Linear Algebra Subprograms, Level 1) Routines," on page 152 lists the Oracle Developer Studio Performance Library BLAS1 routines. No Oracle Developer Studio Performance Library BLAS1 routines are currently parallelized.

TABLE 56 BLAS1 (Basic Linear Algebra Subprograms, Level 1) Routines

| Routine | Function |
| :--- | :--- |
| SASUM, DASUM, SCASUM, <br> DZASUM | Sum of the absolute values of a vector |
| xAXPY |  |
| xCOPY | Product of a scalar and vector plus a vector |
| SDOT, DDOT, DSDOT, SDSDOT, | Dot product (inner product) Quad-precision DQDOTA, DQDOTI available only on |
| CDOTU, ZDOTU, DQDOTA, | SPARC |
| DQDOTI |  |
| CDOTC, ZDOTC | Dot product conjugating first vector |
| SNRM2, DNRM2, SCNRM2, | Euclidean norm of a vector |
| DZNRM2 | Set up Givens plane rotation |
| xROTG | Apply Givens plane rotation |
| SROT, DROT, CSROT, ZDROT | Set up modified Givens plane rotation |
| SROTMG, DROTMG | Apply modified Givens rotation |
| SROTM, DROTM | Index of element with maximum absolute value |
| ISAMAX, IDAMAX, ICAMAX, |  |
| IZAMAX | Scale a vector |
| $x S C A L, ~ C S S C A L, ~ Z D S C A L ~$ | Swap two vectors |
| $x S W A P$ | Compute scaled product of complex vectors |
| CVMUL, ZVMUL |  |

## BLAS2 Routines

Table 57, "BLAS2 (Basic Linear Algebra Subprograms, Level 2) Routines," on page 152 lists the Oracle Developer Studio Performance Library BLAS2 routines. (P) denotes routines that are parallelized.

TABLE 57 BLAS2 (Basic Linear Algebra Subprograms, Level 2) Routines

| Routine | Function |
| :--- | :--- |
| $x$ GBMV | Product of a matrix in banded storage and a vector |
| $x$ GEMV (P) | Product of a general matrix and a vector |
| SGER (P), DGER (P), CGERC | Rank-1 update to a general matrix |
| $(\mathrm{P})$, ZGERC (P), CGERU (P), |  |
| ZGERU (P) |  |
| CHBMV or ZHBMV | Product of a Hermitian matrix in banded storage and a vector |
| CHEMV $(\mathrm{P})$ or ZHEMV (P) | Product of a Hermitian matrix and a vector |


| Routine | Function |
| :--- | :--- |
| CHER (P) or ZHER (P) | Rank-1 update to a Hermitian matrix |
| CHER2 or ZHER2 | Rank-2 update to a Hermitian matrix |
| CHPMV (P) or ZHPMV (P) | Product of a Hermitian matrix in packed storage and a vector |
| CHPR or ZHPR | Rank-1 update to a Hermitian matrix in packed storage |
| CHPR2 or ZHPR2 | Rank-2 update to a Hermitian matrix in packed storage |
| SSBMV or DSBMV | Product of a symmetric matrix in banded storage and a vector |
| SSPMV (P) or DSPMV (P) | Product of a Symmetric matrix in packed storage and a vector |
| SSPR or DSPR | Rank-1 update to a real symmetric matrix in packed storage |
| SSPR2 (P) or DSPR2 (P) | Rank-2 update to a real symmetric matrix in packed storage |
| $x$ SYMV (P) | Product of a symmetric matrix and a vector |
| SSYR (P) or DSYR (P) | Rank-1 update to a real symmetric matrix |
| SSYR2 (P) or DSYR2 (P) | Rank-2 update to a real symmetric matrix |
| $x$ TBMV | Product of a triangular matrix in banded storage and a vector |
| $x$ TBSV | Solution to a triangular system in banded storage of linear equations |
| $x$ TPMV | Product of a triangular matrix in packed storage and a vector |
| $x$ TPSV | Solution to a triangular system of linear equations in packed storage |
| $x$ TRMV (P) | Product of a triangular matrix and a vector |
| $x$ TRSV (P) | Solution to a triangular system of linear equations |

## BLAS3 Routines

Table 58, "BLAS3 (Basic Linear Algebra Subprograms, Level 3) Routines," on page 153 lists the Oracle Developer Studio Performance Library BLAS3 routines. (P) denotes routines that are parallelized.

TABLE 58 BLAS3 (Basic Linear Algebra Subprograms, Level 3) Routines

| Routine | Function |
| :--- | :--- |
| $x$ GEMM (P) | Product of two general matrices |
| CHEMM (P) or ZHEMM (P) | Product of a Hermitian matrix and a general matrix |
| CHERK (P) or ZHERK (P) | Rank-k update of a Hermitian matrix |
| CHER2K (P) or ZHER2K (P) | Rank-2k update of a Hermitian matrix |
| $x$ SYMM (P) | Product of a symmetric matrix and a general matrix |
| $x$ SYRK (P) | Rank-k update of a symmetric matrix |
| $x$ SYR2K (P) | Rank-2k update of a symmetric matrix |
| $x$ TRMM $(\mathrm{P})$ | Product of a triangular matrix and a general matrix |


| Routine | Function |
| :--- | :--- |
| $x$ TRSM (P) | Solution for a triangular system of equations |

## Sparse BLAS Routines

Table 59, "Sparse BLAS Routines," on page 154 lists the Oracle Developer Studio Performance Library sparse BLAS routines. (P) denotes routines that are parallelized.

TABLE 59 Sparse BLAS Routines

| Routines | Function |
| :---: | :---: |
| XAXPYI | Adds a scalar multiple of a sparse vector $X$ to a full vector $Y$. |
| $x$ BCOMM (P) | Block coordinate matrix-matrix multiply. |
| $x$ BDIMM (P) | Block diagonal format matrix-matrix multiply. |
| $x$ BDISM (P) | Block Diagonal format triangular solve. |
| $x$ BELMM (P) | Block Ellpack format matrix-matrix multiply. |
| $x$ BELSM (P) | Block Ellpack format triangular solve. |
| $x$ BSCMM (P) | Block compressed sparse column format matrix-matrix multiply. |
| $x \mathrm{BSCSM}(\mathrm{P})$ | Block compressed sparse column format triangular solve. |
| $x$ BSRMM (P) | Block compressed sparse row format matrix-matrix multiply. |
| $x$ BSRSM (P) | Block compressed sparse row format triangular solve. |
| $x$ COOMM (P) | Coordinate format matrix-matrix multiply. |
| $x$ CSCMM (P) | Compressed sparse column format matrix-matrix multiply |
| $x \operatorname{CSCSM}(\mathrm{P})$ | Compressed sparse column format triangular solve |
| $x$ CSRMM (P) | Compressed sparse row format matrix-matrix multiply. |
| $x$ CSRSM (P) | Compressed sparse row format triangular solve. |
| $x$ DIAMM (P) | Diagonal format matrix-matrix multiply. |
| $x$ DIASM (P) | Diagonal format triangular solve. |
| SDOTI, DDOTI, CDOTUI, or ZDOTUI | Computes the dot product of a sparse vector and a full vector. |
| CDOTCI or ZDOTCI | Computes the conjugate dot product of a sparse vector and a full vector. |
| $x$ ELLMM (P) | Ellpack format matrix-matrix multiply. |
| $x$ ELLSM (P) | Ellpack format triangular solve. |
| xGTHR | Given a full vector, creates a sparse vector and corresponding index vector. |
| xGTHRZ | Given a full vector, creates a sparse vector and corresponding index vector and zeros the full vector. |
| $x$ JADMM (P) | Jagged diagonal matrix-matrix multiply. |


| Routines | Function |
| :--- | :--- |
| SJADRP or DJADRP | Right permutation of a jagged diagonal matrix. |
| $x$ JADSM (P) | Jagged diagonal triangular solve. |
| SROTI or DROTI | Applies a Givens rotation to a sparse vector and a full vector. |
| $x$ SCTR | Given a sparse vector and corresponding index vector, puts those elements into a full |
|  | vector. |
| $x$ SKYMM (P) | Skyline format matrix-matrix multiply. |
| $x$ SKYSM (P) | Skyline format triangular solve. |
| $x$ VBRMM $(P)$ | Variable block sparse row format matrix-matrix multiply. |
| $x$ VBRSM $(P)$ | Variable block sparse row format triangular solve. |

## Sparse Solver Routines

The following tables list routines from SPSOLVE and SuperLU sparse solvers in the Oracle Developer Studio Performance Library. (P) denotes routines that are parallelized.

| TABLE 60 | SPSOLVE Routines |
| :--- | :--- |
| Routines | Function |
| xGSSFS (P) | One call interface to SPSOLVE. |
| xGSSIN | SPSOLVE initialization. |
| xGSSOR | Fill reducing ordering and symbolic factorization. |
| xGSSFA (P) | Matrix value input and numeric factorization. |
| xGSSSL | Triangular solve. |
| xGSSUO | Sets user-specified ordering permutation. |
| xGSSRP | Returns permutation used by solver. |
| xGSSCO | Returns condition number estimate of coefficient matrix. |
| xGSSDA | Deallocate SPSOLVE memory. |
| xGSSPS | Prints solver statistics. |

TABLE 61 SuperLU Routines

| Routine | Function |
| :--- | :--- |
| xgstrf | Computes factorization |
| xgssvx | Factorizes and solves (expert driver) |
| xgssv | Factorizes and solves (simple driver) |
| xgstrs | Computes triangular solve |


| Routine | Function |
| :--- | :--- |
| xgsrfs | Improves computed solution; provides error bounds |
| xlangs | Computes one-norm, Frobenius-norm, or infinity-norm |
| xgsequ | Computes row and column scalings |
| xgscon | Estimates reciprocal of condition number |
| xlaqgs | Equilibrates a general sparse matrix |
| LuSolveTime | Returns time spent in solve stage |
| LUFactTime | Returns time spent in factorization stage |
| LUFactFlops | Returns number of floating point operations in factorization stage |
| LUSolveFlops | Returns number of floating point operations in solve stage |
| xQuerySpace | Returns information on the memory statistics |
| sp_ienv | Returns specified machine dependent parameter |
| xPrintPerf | Prints statistics collected by the computational routines |
| set_default_options | Sets parameters that control solver behavior to default options |
| StatInit | Allocates and initializes structure that stores performance statistics |
| StatFree | Frees structure that stores performance statistics |
| Destroy_Dense_Matrix | Deallocates a SuperMatrix in dense format |
| Destroy_SuperNode_Matrix | Deallocates a SuperMatrix in supernodal format |
| Destroy_CompCol_Matrix | Deallocates a SuperMatrix in compressed sparse column format |
| Destroy_CompCol_Permuted | Deallocates a SuperMatrix in permuted compressed sparse column format |
| Destroy_SuperMatrix_Store | Deallocates actual storage that stores matrix in a SuperMatrix |
| xCopy_CompCol_Matrix | Copies a SuperMatrix in compressed sparse column format |
| xCreate_CompCol_Matrix | Allocates a SuperMatrix in compressed sparse column format |
| xCreate_Dense_Matrix | Allocates a SuperMatrix in dense format |
| xCreate_CompRow_Matrix | Allocates a SuperMatrix in compressed sparse row format |
| xCreate_SuperNode_Matrix | sp_dgemm sp_cgemm |
| sp_preorder | DuperMatrix in supernodal format by a dense matrix |

## Signal Processing Library Routines

Oracle Developer Studio Performance Library contains routines for computing the fast Fourier transform, sine and cosine transforms, and convolution and correlation.

## FFT Routines

Oracle Developer Studio Performance Library provides a set of FFT interfaces that supersedes a subset of the FFTPACK and VFFTPACK routines provided in earlier Oracle Developer Studio Performance Library releases. The old FFT interfaces are included for backward compatibility, and users are encouraged to use the new interfaces. For information on individual FFT routines, see the section 3P man pages.

Table 62, "FFT Routines," on page 157 shows the mapping between the Oracle Developer Studio Performance Library FFT routines and the corresponding FFTPACK and VFFTPACK routines. (P) denotes routines that are parallelized.


| Routine | Replaces | Function |
| :---: | :---: | :---: |
| DFFTZ | DFFTI, DFFTF | Initialize the trigonometric weight and factor tables or compute the one-dimensional forward FFT of a double precision sequence. |
|  | DEZFFTI, DEZFFTF |  |
| DFFTZ2 | DFFT2I | Initialize the trigonometric weight and factor tables or compute the two-dimensional forward FFT of a two-dimensional double precision array. |
|  | DFFT2F |  |
| DFFTZ3 (P) | DFFT3I | Initialize the trigonometric weight and factor tables or compute the three-dimensional forward FFT of three-dimensional double precision array. |
|  | DFFT3F |  |
| DFFTZM | VDFFTI | Initialize the trigonometric weight and factor tables or compute the one-dimensional forward FFT of a set of data sequences stored in a two-dimensional double precision array. |
|  | VDFFTF (P) |  |
| SFFTC | RFFTI, RFFTF | Initialize the trigonometric weight and factor tables or compute the one-dimensional forward FFT of a real sequence. |
|  | EZFFTI, EZFFTF |  |
| SFFTC2 | RFFT2I | Initialize the trigonometric weight and factor tables or compute the two-dimensional forward FFT of a two-dimensional real array. |
|  | RFFT2F |  |
| SFFTC3 (P) | RFFT3I | Initialize the trigonometric weight and factor tables or compute the three-dimensional forward FFT of three-dimensional real array. |
|  | RFFT3F |  |
| SFFTCM | VRFFTI | Initialize the trigonometric weight and factor tables or compute the one-dimensional forward FFT of a set of data sequences stored in a two-dimensional real array. |
|  | VRFFTF (P) |  |
| ZFFTD | DFFTI, DFFTB | Initialize the trigonometric weight and factor tables or compute the one-dimensional inverse FFT of a double complex sequence. |
|  | DEZFFTI, DEZFFTB |  |
| ZFFTD2 | DFFT2I | Initialize the trigonometric weight and factor tables or compute the two-dimensional inverse FFT of a two-dimensional double complex array. |
|  | DFFT2B |  |
| ZFFTD3 (P) | DFFT3I | Initialize the trigonometric weight and factor tables or compute the three-dimensional inverse FFT of three-dimensional double complex array. |
|  | DFFT3B |  |
| ZFFTDM | VDFFTI | Initialize the trigonometric weight and factor tables or compute the one-dimensional inverse FFT of a set of data sequences stored in a two-dimensional double complex array. |
|  | VDFFTB (P) |  |
| ZFFTZ (P) | ZFFTI | Initialize the trigonometric weight and factor tables or compute the one-dimensional forward or inverse FFT of a double complex sequence. |
|  | ZFFTF (P) |  |
|  | ZFFTB (P) |  |
| ZFFTZ2 (P) | ZFFT2I | Initialize the trigonometric weight and factor tables or compute the two-dimensional forward or inverse FFT of a two-dimensional double complex array. |
|  | ZFFT2F (P) |  |
|  | ZFFT2B (P) |  |


| Routine | Replaces | Function |
| :--- | :--- | :--- |
| ZFFTZ3 (P) | ZFFT3I | Initialize the trigonometric weight and factor tables or compute the three-dimensional <br> forward or inverse FFT of three-dimensional double complex array. |
| ZFFT3F (P) |  |  |
| ZFFT3B (P) | Initialize the trigonometric weight and factor tables or compute the one-dimensional forward <br> or inverse FFT of a set of data sequences stored in a two-dimensional double complex array. |  |
|  | VZFFTI |  |

## Fast Cosine and Sine Transforms

Oracle Developer Studio Performance Library fast cosine and sine transform routines are based on the routines contained in FFTPACK (http://www. netlib. org/fftpack/). Routines with a V prefix are vectorized routines that are based on the routines contained in VFFTPACK (http: //www.netlib.org/vfftpack/).

Table 63, "Sine and Cosine Transform Routines," on page 159 lists the Oracle Developer Studio Performance Library sine and cosine transform routines.

TABLE 63 Sine and Cosine Transform Routines

| Routine | Function |
| :--- | :--- |
| COSQB, DCOSQB, VCOSQB, VDCOSQB | Cosine quarter-wave synthesis. |
| COSQF, DCOSQF, VCOSQF, VDCOSQF | Cosine quarter-wave transform. |
| COSQI, DCOSQI, VCOSQI, VDCOSQI | Initialize cosine quarter-wave transform and synthesis. |
| COST, DCOST, VCOST, VDCOST | Cosine even-wave transform. |
| COSTI, DCOSTI, VCOSTI, VDCOSTI | Initialize cosine even-wave transform. |
| SINQB, DSINQB, VSINQB, VDSINQB | Sine quarter-wave synthesis. |
| SINQF, DSINQF, VSINQF, VDSINQF | Sine quarter-wave transform. |
| SINQI, DSINQI, VSINQI, VDSINQI | Initialize sine quarter-wave transform and synthesis. |
| SINT, DSINT, VSINT, VDSINT | Sine odd-wave transform. |
| SINTI, DSINT, VSINTI, VDSINTI | Initialize sine odd-wave transform. |

## Convolution and Correlation Routines

Table 64, "Convolution and Correlation Routines," on page 160 lists the Oracle Developer Studio Performance Library convolution and correlation routines.

TABLE 64

| Routines | Function |
| :--- | :--- |
| xCNVCOR | Computes convolution or correlation |
| xCNVCOR2 | Computes two-dimensional convolution or correlation |

## Miscellaneous Signal Processing Routines

Table 65, "Convolution and Correlation Routines," on page 160 lists the miscellaneous Oracle Developer Studio Performance Library signal processing routines.

TABLE 65
Convolution and Correlation Routines

| Routines | Function |
| :--- | :--- |
| RFFTOPT, DFFTOPT, CFFTOPT, ZFFTOPT | Compute the length of the closest FFT |
| SWIENER or DWEINER | Performs Wiener deconvolution of two signals |
| XTRANS (P) | Transposes array |

See the section 3P man pages for information on using each routine.

## Sort Routines

Table 66, "Sort Routines," on page 160 lists the Oracle Developer Studio Performance Library sort routines.

| TABLE 66 | Sort Routines |
| :--- | :--- |
| Routines | Function |
| BLAS_DSORT (P) | Sorts a real (double precision) vector X in increasing or decreasing order using quick sort <br> algorithm. |
| BLAS_DSORTV (P) | Sorts a real (double precision) vector X in increasing or decreasing order using quick sort <br> algorithm and overwrite P with the permutation vector. |
| BLAS_DPERMUTE (P) | Permutes a real (double precision) array in terms of the permutation vector P, output by <br> DSORTV. |
| BLAS_ISORTV (P) | Sorts an integer vector X in increasing or decreasing order using quick sort algorithm. |
| Sorts a real vector X in increasing or decreasing order using quick sort algorithm and |  |
| overwrite $P$ with the permutation vector. |  |


| Routines | Function |
| :--- | :--- |
| BLAS_SSORT (P) | Sorts a real vector X in increasing or decreasing order using quick sort algorithm. |
| BLAS_SSORTV (P) | Sorts a real vector X in increasing or decreasing order using quick sort algorithm and <br> overwrite P with the permutation vector. |
| BLAS_SPERMUTE (P) | Permutes a real array in terms of the permutation vector P, output by DSORTV. |

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[^0]:    ${ }^{1}$ Memory will be allocated within the routine if the workspace size, indicated by LWORK, is not large enough.

